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Abstract The Human Development Index has experienced substantial modifications in the 2010 edition of the Human Development Report (changes in some of the variables, a different aggregation procedure, and the introduction of distributive considerations, among others). Those changes respond to some well-known shortcomings of the traditional design of this index and entail substantial improvements. There are still some inconsistencies in the new construction that have to be addressed (in particular, the use of a composite variable to approach educational achievements, the use of logs for the income variable and the type of normalization adopted). We discuss in this paper those inconsistencies and suggest some relatively minor changes that would suffice to avoid them.

Key words: Human Development, Inequality, Measurement, Capability Approach, Education

JEL classification, 015, 131

Introduction

Improving human welfare and fostering economic development are two basic goals of most democratic Governments. Evaluating the achievements of policy measures from that perspective requires having *multidimensional indicators* that go beyond the mere computation of the GDP, as pointed out in the recent report by Stiglitz *et al.* (2009). The construction of that type of indicators opens a whole line of research: which are the most relevant dimensions to be considered? How to approximate those dimensions by means of specific variables? How to aggregate those variables into a single indicator?

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The Human Development Index became the most successful multidimensional indicator in the last two decades. It was proposed by the United Nations in 1990 as a protocol to measure the countries' degree of development, based on Amartya Sen's idea of *functionings and capabilities* (see Sen, 1985). This protocol identifies health, education, and material wellbeing as the key human functionings. Traditionally, the achievements in health, education and material wellbeing were associated with the variables *life expectancy at birth*, a mixture of *literacy rate* and *gross enrolment rate* (with weights of 2/3 and 1/3, respectively), and the *log of the per capita GDP*, respectively, suitably normalized. Until the 2010 edition of the Human Development Report (UNDP 2010), the *Human Development Index* (HDI, for short) consisted of the *arithmetic mean* of the normalized values of those three variables.

The twentieth anniversary of the HDI has brought substantial changes in the design of this indicator. Those changes, which have been introduced after an open discussion among the specialists, respond to some well-grounded criticisms to the traditional index.[1] Those criticisms referred to aspects such as: (a) *The number and nature of the selected dimensions*; (b) *The choice of the variables that measure those dimensions*; (c) *The lack of concern for distributive issues*; (d) *The additive structure of the index*; (e) *The lack of theoretical justification of the formula;* and (f) *The strategy of normalizing the raw variables in order to get the partial indices*.

There are three major changes in the new Human Development Index that improve substantially its analytical power:[2]

(i) The substitution of those variables that approach the achievements in education and material wellbeing.

(ii) The use of the geometric mean rather than the arithmetic mean, as a way of aggregating the three selected indicators.

(iii) The introduction of distributive considerations by means of a new complementary index.

In spite of the improvements that this new index incorporates, there are still some arguable aspects of its construction worth revising. On the one hand, from a purely theoretical viewpoint, there are some flaws in the design of the index that should be addressed. On the other hand, the HDI 2010 has received a number of criticisms related to the apparent performance of African countries and the questionable trade-offs between variables, according to the new methodology (Ravallion, 2010).

We shall discuss here both the changes introduced in the 2010 version of the HDI, the remaining shortcomings, and some simple ways to create further improvements to this development index. It is worth noting from the outset that those changes are partly interdependent, so that decisions on one of them will have some bearing on decisions on the others. Our proposals here are constructed under the self-imposed restriction of using the data already available in the report.[3]

The rest of the paper goes as follows. Sections 2 and 3 discuss the choice of variables and the aggregation formula, respectively. Section 4 revises some of the methodological aspects of the new construction (the use of logs, the normalization strategy, the introduction of inequality measures). Section 5 deals with the concern of inequality. Section 6 summarizes the changes we propose and compares the results so obtained with those corresponding to the HDI 2010. A few final comments close the paper.

The choice of variables

Assuming that health, education and material wellbeing are the three key dimensions to evaluate human development, one has to choose the right variables that approach the achievements in those dimensions. What is the best indicator depends on the aspects we want to capture, something that involves the very notion of human development we adopt, and on data availability, always a limiting factor.

The HDI 2010 has introduced two changes in those variables. One refers to the variable that approaches the achievements in material wellbeing and the other to the way of measuring educational achievements. Life expectancy at birth has been kept as the variable that approaches the health dimension.[4]

The per capita GDP has been substituted by the per capita *Gross National Income*, in order to take into account the outcomes of the nationals living abroad and the proceeds of the firms operating in other countries. This is a relatively minor change that nevertheless may be regarded as an improvement.

Much more important is the change in the variable that measures educational achievements. The former index of education used in the traditional HDI did not reflect the differences in human capital (particularly in highly developed countries), mostly due to the excessive weight given to the literacy rate. There are several ways of constructing a better indicator of educational achievements.[5] The HDI 2010 has chosen a composite variable: the geometric mean of 'mean years of education' (adults) and 'expected years of schooling' (children), suitably normalized. Even though this change is a major improvement with respect to the former index, it involves two significant shortcomings. On the one hand, the combination of those two variables makes less transparent the analysis on their impact in the human development index (in particular it is not easy to interpret a variable that is neither an expected value, as life expectancy, nor an average, as the per capita GNI). On the other hand, using the geometric mean to compute the partial indicator of education is hard to justify, as the geometric mean fosters equalizing the values of the involved variables; this amounts to saying that improving the educational achievements of the children with respect to the adults is partly penalized due to the increment in the dispersion of the variables.

We believe that choosing one of the two basic variables, the *expected years of schooling*, is a much better choice. There are several reasons for this. First, it simplifies the interpretation of the aggregate index. Second, it avoids having to decide (and justify) the relative weights of both components and the aggregation formula. Third, it focuses on the future (possibilities) rather than on the past (realizations). Fourth, it is an incentive to educational investment, as it gets reflected in the index much more rapidly. And fifth, it seems the choice that is consistent with the use of life expectancy at birth to approach the health dimension.[6]

The aggregation formula

The geometric mean

The use of the arithmetic mean as a way of aggregating the three partial indices was a notorious source of discomfort, in spite of the obvious fact that it was a simple and intuitive transformation. The reason was threefold. First, because such an additive structure implies assuming perfect substitutability between their components (linear indifference curves), a feature hard to justify. Second, because the index generated a ranking sensitive to the normalization of the different variables, as changing the normalization amounted to modifying the weights with which those variables entered the index. And third, because there was no theoretical justification supporting such a choice.

The use of some non-linear aggregator, instead of the arithmetic mean, with a proper theoretical support, was an obvious and long claimed request. The geometric mean is a sensible choice, among other options, as it exhibits much better properties. In particular, it corrects the perfect substitutability feature, penalizes the dispersion of the variables that are aggregated, and it is a relatively easy concept that may substitute the arithmetic mean and still keep an intuitive interpretation that many people can understand. See Chakravarty (2003), Foster *et al.* (2005), Herrero *et al.* (2010a), Seth (2009, 2010), among others, for a discussion.

The choice of the geometric mean is certainly an important improvement. Yet, the Report does not provide any theoretical justification of the index (an axiomatic support, to be precise). This is not only an aesthetic question. Besides eliminating the arbitrary nature of the choice, the characterization permits one to give the index a true cardinal dimension and then allows performing comparisons on *how much* the human development has changed. Without such a justification it is not clear what the index allows for.

We propose here an elementary characterization of the geometric mean that fits well within this context and permits one to justify this aggregation method.[7]

A pinch of theory: Axiomatization of the new index

We follow here the *axiomatic method* in order to justify the choice of the aggregation formula. That is, we identify the proposed indicator with a unique set of intuitive properties: *neutrality, scale*, and *ratio consistency.* Let us formalize these ideas.

Suppose that we want to define an evaluation index for a given society that aggregates three (normalized) variables. We define a **social state** as a vector (α, β, γ) with three components, each of which belongs to the interval [0,1] (that is, the values of those characteristics are already normalized so that the differences in their mean values have been cancelled). Therefore, $\Omega = [0, 1]^3$ is the space of admissible social states.

A **Social Evaluation Index** is a continuous single-valued mapping $I : \Omega \to \mathbb{R}$ that provides a numerical evaluation of social states.

We first introduce two basic requirements on the social evaluation index: neutrality and scale. *Neutrality* makes it explicit that all characteristics enter the evaluation function on an equal foot. *Scale* fixes the value of the index when the social state is uniform (i.e. all entries are identical), by choosing precisely that very same value. Formally:

- **Neutrality.** For each point $(\alpha, \beta, \gamma) \in \Omega$, if $\pi(\alpha, \beta, \gamma)$ denotes a permutation of its elements, then $I[\pi(\alpha, \beta, \gamma)] = I(\alpha, \beta, \gamma)$.
- **Scale**. Let $p \in [0,\ 1]$. Then, $I(p, p, p) = p$.

The last property, *Ratio Consistency*, requires that the relative value of the indices of two social states with a common component does not depend on the value of that common component. Formally:

- **Ratio Consistency.** Let $(\alpha, \beta, \gamma),\ (\alpha, \beta', \gamma') \in \Omega$ be two strictly positive social state vectors, with the same first component α. If that common component changes to a different one, α', the ratio of the associated indices does not change. That is,

$$\frac{I(\alpha, \beta, \gamma)}{I(\alpha, \beta', \gamma')} = \frac{I(\alpha', \beta, \gamma)}{I(\alpha', \beta', \gamma')}$$

This property says the following. Suppose that countries A and B have the same values concerning the health variable and different values with respect to education and material wellbeing, all positive, so that the overall index of country A is twice that of country B. Now both countries experience an improvement in health that changes the corresponding variable by exactly the same amount. That change obviously alters the associated development indices. Ratio Consistency implies that the new index of country A is still twice the new index of country B. That is to say, the relative value of the index is not affected by an equal change of a common value of a given variable.

Remark: *We ask this property to hold just for one component (the first one, in our definition) for the sake of parsimony. When combined with 'neutrality' this property actually applies to any common value.*

Note that this consistency requirement is cardinal in nature and involves a separability feature in the evaluation index.

The following result is obtained:

Theorem: *A social evaluation index I(.) satisfies neutrality, scale and ratio consistency, if and only if it takes the form:*

$$I(\alpha, \beta, \gamma) = \alpha^{1/3}\beta^{1/3}\gamma^{1/3}$$

Moreover, those properties are independent.

Proof:

First note that scale implies $I(\alpha, \alpha, \alpha) = \alpha$, for all $\alpha \in [0, 1]$. By neutrality, the property of ratio consistency can be applied to any component of vector (α, β, γ).

Take now a vector$(\alpha, \beta, \gamma) >> 0$. By ratio consistency and neutrality we can write:

$$\frac{I(\alpha, \alpha, \alpha)}{I(\alpha, \alpha, \beta)} = \frac{I(\alpha, \alpha, \alpha)}{I(\beta, \alpha, \alpha)} = \frac{I(\alpha, \beta, \gamma)}{I(\beta, \beta, \gamma)}$$

And thus,

$$I(\alpha, \alpha, \beta) = \alpha\frac{I(\beta, \beta, \gamma)}{I(\alpha, \beta, \gamma)}$$

In a similar way, $\frac{I(\beta, \beta, \beta)}{I(\beta, \beta, \gamma)} = \frac{I(\alpha, \beta, \gamma)}{I(\alpha, \gamma, \gamma)}$, so that $I(\beta, \beta, \gamma) = \beta\frac{I(\gamma, \gamma, \alpha)}{I(\alpha, \beta, \gamma)}$, and $I(\alpha, \alpha, \beta) = \alpha\beta\frac{I(\gamma, \gamma, \alpha)}{[I(\alpha, \beta, \gamma)]^2}$

Finally, $I(\gamma, \gamma, \beta) = \gamma\frac{I(\alpha, \alpha, \beta)}{I(\alpha, \beta, \gamma)}$. Substituting, $I(\alpha, \alpha, \beta) = \alpha\beta\gamma\frac{I(\alpha, \alpha, \beta)}{[I(\alpha, \beta, \gamma)]^3}$.

Therefore, we conclude:

$$I(\alpha, \beta, \gamma) = \alpha^{1/3}\beta^{1/3}\gamma^{1/3}$$

(ii) To separate the properties let us consider the following indices:

(1) $I(\alpha, \beta, \gamma) = \frac{1}{3}(\alpha + \beta + \gamma)$. It satisfies neutrality, scale but not ratio consistency.

(2) $I(\alpha, \beta, \gamma) = \frac{\alpha\beta\gamma}{3}$. It satisfies neutrality, ratio consistency and not scale .

(3) $I(\alpha, \beta, \gamma) = \alpha^a\beta^b\gamma^c$, with $a + b + c = 1$ and not all of them are equal. It satisfies ratio consistency and scale but not neutrality. **Q.e.d.**

This theorem says that, among those indices that satisfy neutrality and scale, ratio consistency determines that the evaluation formula is given by the geometric mean of the corresponding normalized values of the chosen variables.

It is worth noting that if we change the units of any dimension in the normalization of its component, not only the ranking of the countries is preserved but also the relative size of the indices.

Logs, max-min values, substitution rates, and other beasts

The HDI 2010 has opened an important discussion about some methodological issues, which refer to the internal structure of the index. Namely, using logs for the income measure, the choice of the normalization formula, the implications of those choices in terms of marginal rates of substitution, etc. See the discussions in the blog 'Let's Talk Human Development'; see also Herrero *et al*. (2010b), Ravallion (2010), and Zambrano (2011).

To log or not to log… That is (one of) the question(s)

The use of logs is usually introduced as a way of recognizing that the effect of one additional unit of a given variable decreases with the level at which this happens (the increment of the function is proportional to the relative increment of the variable). The rationale for using logs in this context is linked to the notion of development we try to capture with our index. The refurbishing of the HDI has put this question on the table once again: what is the nature of the human development index?

If we think of the HDI more as a *welfare measure*, it might be reasonable to keep measuring all three variables in terms of logs (or in terms of any increasing and concave function) as those values can be interpreted as utility measures of an average citizen. The principle of 'decreasing marginal utility' should be applied not only to income but also to health and education.

If we rather think of the HDI as an indicator that provides a summary description of the capacity of a country to grow, compete and enhance material wellbeing, as we actually do, the use of logs does not seem justified in any of its variables. Indeed, its use helps conceal the existing differences and has doubtful implications on the substitution rates.

There is still an intermediate approach that can be invoked to support the choice made in the 2010 Human Development Report. In this one considers that the HDI is neither a welfare index nor a pure descriptive measure, but rather an estimate of *capabilities*. This would agree with the original approach to human development by the United Nations. According to this interpretation, the variables that approach the achievements in health and education can be regarded as direct indicators of the corresponding capabilities, whereas the variable that approaches material wellbeing is only an indirect measure, as income itself is not a capability (see Zambrano, 2011) for a formalization of those ideas). This amounts to saying that there are two types of variables to consider when constructing the human development index: direct and indirect indicators of capabilities.

Even though this is a consistent approach that allows justifying the present format of the HDI, it involves some drawbacks. First, it does not fit very well with some of the other novelties of the HDI 2010 (especially with respect to inequality adjusted measures, discussed below). Second, it imposes restrictions on the normalization formula, as one cannot take a min value equal to zero for the logged variable. Third, it has a relevant impact on the substitutability of the primary variables whose meaning is not very clear (or, alternatively, prevents making sensible calculations of marginal rates of substitution).

Let us note, in passing, that interpreting the per capita GNI as an indirect measure of capability is compatible with alternative ways of constructing the index of material wellbeing.[8]

The normalization formula

The normalization formula chosen by the HDI 2010 keeps the use of goalposts for max and min values. Namely, transforming the original values into *relative gains*, by subtracting a min value and dividing by some fixed range of the variable.[9] This formula, which is one of the standards, has three negative implications worth considering.

First, it makes the whole construction of the HDI dependent on the arbitrary choices of the normalization parameters (in particular, on the min values). This is quite unpleasant, to say the least. The fact that changing the min values can revert the ranking and modify the relative valuations is very bad news. Indeed, the dependence of the ranking on the arbitrary choice of normalization values was one of the main criticisms with respect to the arithmetic mean. Note that this criticism does not apply to the caps (the max values used to define the range), because a change of the max values only affects the units of measurement and therefore they cannot alter the ranking. To see this note that a normalized value, h say, can be expressed as:

$$h = \frac{H - H_{\min}}{H_{\max} - H_{\min}} = \alpha(H - H_{\min})$$

where H is the value of the original variable. A change in $H_{\max}$ only alters the coefficient α (the units of measurement) for all countries and therefore cannot change the ranking. A change on $H_{\min}$, on the contrary, affects both the units and the net value of the variable so that it may well change the ordering of the countries.[10]

Second, deducting any positive value from the original variables worsens the picture we get of those countries with a lower performance.

And third, the use of min values in the normalization has an extra impact on the marginal rates of substitution. We all know how a Cobb-Douglas function behaves when approaching zero values in one of its components. Therefore, subtracting whatever amount to an already very close to zero magnitude will increase substantially (and artificially) the associated marginal rates of substitution.

Those shortcomings bring us to propose the use of a different normalization formula. Namely, substituting the original values for its shares with respect to some reference values chosen in order to facilitate the interpretation of the partial indices. That is, we substitute each individual variable $x \in [x_{\min}, x^{\max}]$, by $x' = \frac{x}{x*}$, for some reference value x^*. This process simply defines implicitly the units in which we measure the variable, and affects neither the ranking that the HDI produces nor the relative valuations of any two countries, nor the associated marginal rates of substitution. When we consider a period of time, as the 2010 Report does, we can take a suitable reference value for the whole period. The closest alternative to what has been done is to take the share of some maximum value, $x* = \mathrm{x}^{\max}$. Let us insist on the fact that the choice of the reference variable affects neither the ranking nor the relative valuation of any two countries.

The concern for equality: inequality-adjusted income

The lack of concern for distributive issues, most specifically with respect to the income dimension, was one of the most surprising features of the traditional HDI. Even more so considering that there are statistics on income inequality for most of the countries and we have a well-established theory that permits one to link the evaluation of the size and the distribution of income. Moreover, there are a number of contributions that suggest ways of introducing equality concerns in this particular context (Anand and Sen (1994b), Hicks (1997), Foster, López-Calva and Székely (2005), Herrero *et al*. (2010 a, b), Seth (2009), (2010)).

The standard way of conveying a normative content to an inequality measure is that of interpreting inequality as a welfare loss, in the tradition of Dalton, Atkinson, Sen and Kolm, to name a few representative thinkers. To do so we identify the inequality adjusted mean of an income distribution $\mathbf{y} = (y_1, y_2, ..., y_n)$ with the ***egalitarian equivalent income***, y^e (i.e. the amount of income that equally distributed would yield the same social welfare as the current income distribution). This value can in turn be expressed as:

$$y^e = \mu(\mathbf{y})\left[1 - I(\mathbf{y})\right]$$

where $\mu(\mathbf{y}),\ I(\mathbf{y})$ stand for the per capita income and an inequality measure, respectively. This formula tells us that the inequality adjusted mean income corresponds to the per capita income deflated by an inequality measure. The term $\mu(\mathbf{y}) \times I(\mathbf{y})$ is a measure of the penalty due to the unequal income distribution. The choice of the right inequality index can be done making use of the properties we deem relevant (e.g. decomposability, degree of preference for equality, etc.).[11]

This construction is based on two key conceptual elements. First, the inequality measure refers precisely to the variables that are used to get the mean value. Second, perfect equality is the best possible world.

The HDI 2010 incorporates distributive concerns by providing a new and separate index: the *Inequality Adjusted Human Development Index*, IHDI, in which each of the variables is adjusted by inequality. Unfortunately, the IHDI does not satisfy the conceptual requirements pointed out above. Let us see why, by reviewing each inequality adjusted variable.

The use of the egalitarian equivalent income, to introduce distributive considerations in the evaluation function, fits neither with the use of logs nor with the normalization choice. If we measure inequality over the income distribution vector, we cannot use consistently the log of income as the reference variable. Moreover, the normalization used (with or without logs) is also inconsistent, as the chosen inequality measure is sensitive to the choice of min values in the normalization.[12] If we want to keep the interpretation of the capability approach proposed in Zambrano (2011), it seems that one should measure the inequality over the vector of log income values. Be as it may, the inconsistency with the normalization choice remains, unless one measures the inequality over the distribution of the normalized individual logged variables. But the meaning of that exercise is far from clear.

Interestingly enough, the egalitarian equivalent income can be interpreted in terms of the opportunity faced by an agent that lands in the corresponding country. That opportunity corresponds to the discounted value of the expected income, where discounting refers to the risk associated to the distribution pattern. Alternatively, one can think of this variable as a capability measure that transforms income into material wellbeing (so as an alternative to the use of logs within this approach).

The inequality adjusted education index is hard to justify. On the one hand, because it is a combined measure. On the other hand, because the data used to calculate inequality correspond to a different dataset. So methodologically this is a rather weak construction.

As for the inequality adjusted health measure, note that it is a rather complex concept, difficult to construct and to understand (see Kovacevic (2010) for an explanation of its construction). The mortality experienced by the population is captured in the age-specific mortality rates. The inequality in health is the inequality in expected life lengths obtained from the mortality rates. To assess inequalities in a variable that is essentially an expectation, the historical inequality in the age of death is taken as a proxy. Furthermore, there is no relationship with any sort of socioeconomic stratification of the population, a fact that is, in general, considered as extremely relevant in the analysis of inequality of opportunity in health.

Notice that, due to the lack of data in a significant number of countries, the inequality index both for education and health are estimates with no common methodological support.

We propose here to use the inequality adjusted income, for an income without logs and a consistent normalization (i.e. in terms of shares and not in terms of relative gains) as the variable to capture the wellbeing capabilities in the HDI. As for the dimensions regarding health and education, unless we find a better way of approaching inequality, we believe it is more convenient to skip distributive considerations on those aspects.

The type of inequality index chosen to define the egalitarian equivalent income is a matter of discussion, but it should not be a major problem. The use of the geometric mean of individual incomes, proposed initially in Foster, López-Calva and Széleky (2005), is a nice choice that preserves the symmetry with respect to the construction of the HDI. The Gini index is another sensible alternative, as it is a very familiar inequality index and we have data built basically on the same type of variables used to approach the per capita GNI. The Theil index is still another alternative, that becomes preferable when decomposing inequality is a relevant part of the analysis.

The Newer HDI *vis à vis* the HDI 2010

The discussion presented in former sections points out that there are better ways of approaching the measurement of human development, while keeping the basic principles of the HDI 2010. Our proposal can be summarized as follows:

(i) Concerning the definition of the variables:
   a. Use the expected years of schooling instead of a combined variable, to give more weight to the future capacities and to simplify the interpretation of this partial indicator; and
   b. Measure material well-being in terms of the egalitarian equivalent per capita GNI without logs, to avoid inconsistencies and undesired side-effects, and taking distributional aspects into account in a consistent manner.

(ii) Concerning the normalization process, all variables are to be normalized in terms of shares of some max values. This strategy ensures an easy interpretation of the normalized values and, most importantly, makes the resulting ranking of the countries, the marginal rates of substitution, and the pair-wise comparison of relative achievements independent on the normalization parameters.

(iii) Concerning the aggregation formula, we find that the geometric mean is the right choice, once it has been justified on theoretical grounds.

As for the reference values used in our normalization, we take 83.2 years for life expectancy at birth, 20.6 years for expected years of schooling and 60,000 US $ for the inequality adjusted mean income. The first two values

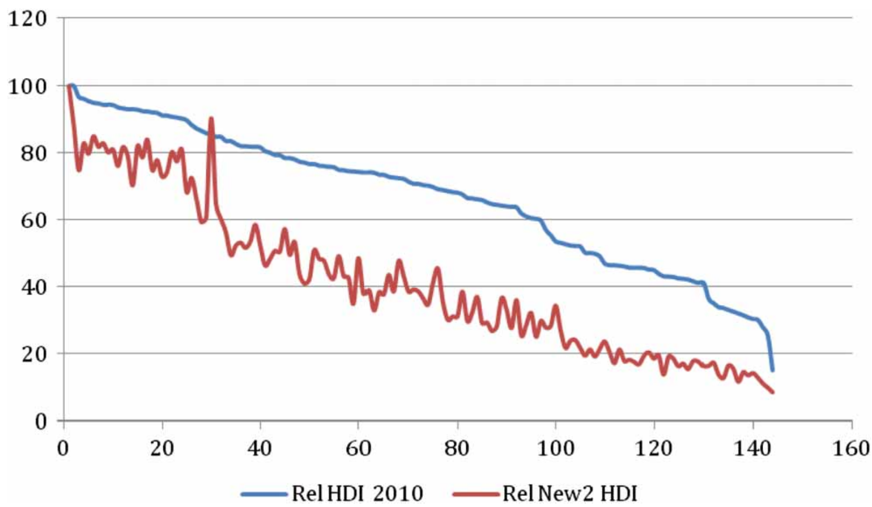

FIGURE 1. The relative values of the HDI 2010 and the Newer HDI (2010)
Source: See Table A1 in the Appendix

correspond to the original caps in the HDI 2010 whereas the third one is a reasonable maximum value for the egalitarian equivalent income. All together, the HDI 2010 is defined as follows:

$$\text{Newer HDI} = \left(\frac{x_{\text{health}}}{83.2} \cdot \frac{x_{education}}{20.6} \cdot \frac{x_{\text{income}}(1-G)}{60000}\right)^{\frac{1}{3}}$$

where x_{health} is the life expectancy at birth, $x_{\text{education}}$ are the expected years of schooling, x_{income} is the GNI per capita, and G is the Gini index.

Figure 1 plots the relative values of the HDI 2010 and our proposal (the Newer HDI). The maximum values of both indices are set equal to 100 and the remaining values scaled down accordingly. Countries are ordered following the HDI 2010 rank, so that the line monotonously decreasing describes the world distribution of that variable. The non-monotonous line corresponds to the values of our proposed index without altering the ordering of the countries in the horizontal axis. Therefore, the peaks that appear correspond to changes in the ranking with respect to the 2010 HDI. Note that the average slope of that second line tells us that the world distribution of the Newer HDI is more unequal than that of the Newer HDI (due to the impact of unlogged income values).

Another way of expressing the relevance of the change in the ranking induced by this new way of measuring human development is by noting that 18% of the countries in the report change 10 or more positions in 2010. Qatar and Gabon are the extreme cases of improvement, advancing 28 and 22 positions, respectively. Georgia, Belize, New Zealand and Peru are the extremes on the opposite side. They go down the ranking by 20, 20, 18 and 15 positions, respectively.

Table A1 in the Appendix provides all the data.

As for the substitution rates concerning health and income, it can be checked that with our formula those rates reduce more than 70%, on average, with respect to those corresponding to the HDI 2010 (see Ravallion (2010) and the tables in the Appendix). In particular, the marginal rate of substitution between health and income (the monetary valuation of an extra year of life), is governed by the simple and intuitive formula $\frac{x_{income}}{x_{health}}$, where x_{income} and x_{health} are the original values of mean income and life expectancy at birth, respectively, of the corresponding country.[13] Consequently, this marginal valuation is independent of the normalization and corresponds to the average income per year of life to be lived. Similarly, we can calculate the monetary valuation of an extra year of schooling as the ratio between per capita GNI and the expected years of schooling (notice that having a single variable makes much easier the interpretation of this valuation).

Ravallion (2010) also criticizes the extreme differences observed in the HDI 2010 between countries with very similar raw data (he singularizes the case of the last two countries, Zimbabwe and Democratic Republic of the Congo, whose HDI 2010 are 0.140 and 0.239, respectively). This problem does not appear in our formulation (where we find values of 0.084 and 0.072 for Zimbabwe and Democratic Republic of the Congo, much more similar than those corresponding to the HDI 2010).

Final comments

We have proposed here additional changes in the HDI 2010 in order to avoid some inconsistencies that the present format involves. We have imposed clear boundaries to our assessment exercise. On the one hand, we keep the basic approach to the measurement of human development, which is associated with three essential dimensions that are deemed equally important: health, education and material wellbeing. On the other hand, we assume that the final formula should not use additional data with respect to those included in the 2010 report.

Without those limits there is still scope for the discussion with respect to the relevant dimensions, the variables that measure the achievements in those dimensions, and the weights with which they should enter the formula. In Herrero *et al.* (2010 b) we propose a HDI specific for the OECD countries, changing life expectancy for life potential and expected years of schooling for expected years of education between 15 and 29 (which is a measure of non-compulsory education). Those changes aim at discriminating better the actual situation of those countries with a similar degree of development. The elaboration of those ideas is left for the future.

It is worth noting that some of the criticisms received by the aggregation formula introduced in the HDI 2010 are not well founded. The problem is not the geometric mean but rather the use of logs for the income variable and the way in which the raw variables are normalized.

Our proposal introduces small changes in the HDI 2010 formula that nevertheless are sufficient to avoid most of its remaining shortcomings. The normalization proposed here does not affect the ranking of the countries, their relative evaluation, or the value of the marginal rates of substitution. Moreover, using the income variable without logs permits one to consistently incorporate distributive aspects in the evaluation of material wellbeing. Finally, the Newer HDI allows for truly consistent cardinal comparisons among the countries.

Note that dispensing with the logs in the measurement of material wellbeing has a relevant impact on the ranking and the relative evaluation of the countries. This is so because the world distribution of the per capita GNI is much more asymmetric than the other components.[14] That explains the significant jump upwards of Qatar, as it is a country with a per capita GNI much higher than any other one for very specific reasons. It is therefore to be treated more as an outlier than as a reference to draw conclusions about the significance of the resulting changes in the ranking. Be that as it may, we understand that making more evident the differences in the countries' achievements is a plus rather than a minus, besides avoiding the inconsistencies that have been pointed out, and that it yields a more accurate overall picture of the differences in human development.

That being said, one can reasonably argue that what is an advantage from a descriptive point of view may become a disadvantage if we think of the HDI as a guide to implementing development policies (i.e. inducing the governments to pay more attention to the non-income dimensions of wellbeing). To avoid this side-effect and yet keep the consistency in the design of the index one should put a different weight to the income dimension (smaller than 1/3) rather than reverting to the use of logs. That is also a matter for future research.

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Notes

1 See the discussion in the UNDP Research Papers' Series. See the criticisms in Anand and Sen (1994 a, b); Hicks (1997); Sagar and Najam (1999); Osberg and Sharpe (2002); Phillipson

and Soares (2001); Pinilla and Goerlich (2005); Foster *et al.* (2005); Becker *et al.* (2005); Stiglitz *et al.* (2009); Herrero *et al.* (2010a, b).

2 Besides those methodological changes there are also some other relevant novelties, such as the reconstruction of the evolution of those variables since 1980 in a time consistent series and the introduction of a brand new multidimensional poverty measure.

3 This paper is built over a former contribution (Herrero *et al*, 2010b), written before the HDI 2010 appeared. Since some of our suggestions there have already been incorporated to the New HDI, we shall not insist on them.

4 Herrero *et al.* (2010b) propose to change life expectancy at birth by *life potential*, in order to get a better estimation of development capabilities, in the case of highly developed countries. This is so because life expectancy at birth is a variable that is independent on the demographic structure. As a consequence, it tends to over-weight the health component of those countries with a higher share of old people. That aspect is most arguable in the context of evaluating development capabilities, for it ignores the size of the present and future working age population. Be as it may, we leave aside here this question, bearing in mind that we aim at proposing some changes in the index without recurring to new data. Moreover, data on life potential are only available for OECD countries.

5 One is to add up all students that reach some reference level, as this implies that those with higher levels of studies are computed several times (for example, a person that gets a college degree weights three times more than a person that left studies after primary school); this is the approach followed by the American Human Development Index. (see Burd-Sharps, 2008). An alternative way is computing the average number of years of school attendance or the percentage of working age population with non-compulsory studies (as in Herrero *et al.* 2010c). A sensible option for highly developed countries is to take the 'expected years of education between 15 and 29' used by the OECD when assessing education and the labour market (Herrero *et al*. 2010b).

6 A natural way of interpreting the capabilities in the health and education variables is by means of the expectations of newcomers in the society. This is indeed the philosophy behind the selection of life expectancy at birth. Note that choosing mean years of education of adults would be more in line with the selection of life potential to measuring actual achievements.

7 Needless to say the geometric mean is just a special case of the generalized means, which have been already characterized (e.g. Foster *et al.*, 2005; Seth, 2009). Our contribution here is that of presenting a very simple and self-contained characterization result. An alternative characterization appears in Herrero *et al.* (2010a).

8 A case in point is that in which material wellbeing is approached in terms of the egalitarian equivalent income (see below).

9 The min values chosen are: 20 years for life expectancy at birth, 0 for both education variables, and $163 (the lowest value attained for any country in recorded history, in Zimbabwe, 2008). As for the max values, we find: 83.2 years for life expectancy, 13.2 for mean years of schooling, 20.6 for expected years of schooling, and $108,211 for the per capita GNI.

10 To assess the empirical extent of this aspect we have calculated the HDI 2010 by normalizing the raw variables as shares to the same max values used in the report and keeping everything else the same. The result is that, in 2010, some 30% of the countries change their ranking by five or more positions and six countries change 10 positions or more.

11 See, for instance, Cowell (1995), Sen and Foster (1997), Goerlich and Villar (2009).

12 We consider here the family of relative inequality measures, which is the one taken as reference in the report.

13 Under the implicit assumption that inequality remains constant.

14 Just to give a hint: the 2010 figures show that the coefficient of variation of the per capita GNI is more than seven times that of life expectancy variable and more than four times that of the education index. Concerning the extreme values, the country with the highest life

expectancy value was about twice that of the lowest one; the country with the highest education index was about ten times the lowest one; and the country with the highest per capita GNP was some 450 times the lowest one!

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Appendix

TABLE A1. The Human Development Index and its components

Country	Life expectancy at birth (years)	Expected years of schooling (years)	GNI per capita (PPP US$)	Gini (%)	Newer HDI	HDI 2010	Ranking HDI 2010	Ranking Newer HDI	Difference in rankings	Cost of an extra year in health (PPP US$)	Cost of an extra year in education (PPP US$)
Norway	81.01	17.32	58809.53	25.8	0.841	0.938	1	1	0	725.99	3395.05
Australia	81.86	20.46	38691.71	35.2	0.742	0.937	2	3	−1	472.69	1891.33
New Zealand	80.60	19.70	25437.50	36.2	0.631	0.907	3	21	−18	315.59	1290.93
United States	79.58	15.75	47093.85	40.8	0.698	0.902	4	7	−3	591.75	2991.03
Ireland	80.33	17.86	33077.57	34.3	0.672	0.895	5	15	−10	411.79	1852.44
Netherlands	80.33	16.71	40657.78	30.9	0.716	0.890	6	4	2	506.13	2432.95
Canada	81.04	15.95	38668.37	32.6	0.689	0.888	7	9	−2	477.15	2424.20
Sweden	81.27	15.56	36936.27	25.0	0.698	0.885	8	6	2	454.48	2374.24
Germany	80.21	15.59	35308.04	28.3	0.675	0.885	9	14	−5	440.18	2265.32
Japan	83.17	15.07	34692.46	24.9	0.682	0.884	10	12	−2	417.15	2302.58
Korea (Republic of)	79.80	16.82	29517.62	31.6	0.641	0.877	11	20	−9	369.89	1754.67
Switzerland	82.20	15.47	39849.09	33.7	0.689	0.874	12	10	2	484.76	2576.25
France	81.65	16.15	34340.71	32.7	0.667	0.872	13	16	−3	420.60	2126.73
Israel	81.15	15.56	27831.34	39.2	0.592	0.872	14	26	−12	342.95	1788.45
Finland	80.09	17.12	33871.73	26.9	0.691	0.871	15	8	7	422.94	1978.06
Belgium	80.30	15.94	34872.70	33.0	0.663	0.867	16	17	−1	434.31	2187.83
Denmark	78.71	16.87	36404.41	24.7	0.707	0.866	17	5	12	462.50	2158.31
Spain	81.28	16.37	29661.16	34.7	0.630	0.863	18	22	−4	364.94	1812.35
Hong Kong. China (SAR)	82.51	13.77	45090.48	43.4	0.656	0.862	19	18	1	546.46	3274.71
Greece	79.69	16.48	27580.38	34.3	0.614	0.855	20	24	−4	346.09	1673.97
Italy	81.44	16.27	29619.21	36.0	0.625	0.854	21	23	−2	363.69	1820.23
Austria	80.40	15.04	37055.90	29.1	0.676	0.851	22	13	9	460.92	2463.07
United Kingdom	79.78	15.94	35087.16	36.0	0.652	0.849	23	19	4	439.83	2201.10
Singapore	80.72	14.40	48893.19	42.5	0.682	0.846	24	11	13	605.68	3395.36
Czech Republic	76.93	15.16	22678.39	25.8	0.576	0.841	25	27	−2	294.78	1496.07
Slovenia	78.80	16.74	25857.03	31.2	0.611	0.828	26	25	1	328.13	1544.37

Slovakia	75.12	14.86	21657.78	25.8	0.559	0.818	27	28	−1	288.29	1457.25
Estonia	73.74	15.83	17167.68	36.0	0.500	0.812	28	32	−4	232.82	1084.16
Hungary	73.88	15.27	17472.12	30.0	0.512	0.805	29	30	−1	236.51	1144.19
Qatar	75.97	12.74	79426.35	41.1	0.761	0.803	30	2	28	1045.55	6235.79
Portugal	79.09	15.49	22105.19	38.5	0.545	0.795	31	29	2	279.50	1426.70
Poland	76.00	15.24	17803.06	34.9	0.507	0.795	32	31	1	234.24	1167.96
Lithuania	72.08	15.96	14823.72	35.8	0.474	0.783	33	35	−2	205.66	929.07
Chile	78.79	14.53	13561.02	52.0	0.417	0.783	34	46	−12	172.12	933.44
Argentina	75.70	15.53	14603.33	48.8	0.441	0.775	35	40	−5	192.92	940.11
Latvia	73.01	15.38	12944.18	36.3	0.448	0.769	36	38	−2	177.28	841.76
Montenegro	74.57	14.44	12490.82	36.9	0.435	0.769	37	41	−4	167.51	865.02
Romania	73.21	14.82	12843.70	32.1	0.451	0.767	38	36	2	175.44	866.74
Croatia	76.69	13.84	16388.59	29.0	0.493	0.767	39	33	6	213.71	1184.13
Uruguay	76.68	15.70	13808.44	47.1	0.441	0.765	40	39	1	180.08	879.70
Panama	75.96	13.45	13346.85	54.9	0.391	0.755	41	53	−12	175.70	992.27
Mexico	76.69	13.44	13971.41	51.6	0.408	0.750	42	49	−7	182.18	1039.34
Malaysia	74.73	12.47	13926.86	37.9	0.428	0.744	43	43	0	186.36	1117.27
Bulgaria	73.75	13.67	11139.16	29.2	0.426	0.743	44	44	0	151.04	814.97
Trinidad and Tobago	69.93	11.42	24233.27	40.3	0.483	0.736	45	34	11	346.56	2121.84
Serbia	74.38	13.45	10449.37	28.2	0.418	0.735	46	45	1	140.49	776.86
Belarus	69.63	14.61	12925.70	28.8	0.450	0.732	47	37	10	185.65	884.56
Costa Rica	79.10	11.73	10869.63	48.9	0.369	0.725	48	55	−7	137.42	926.82
Peru	73.67	13.81	8424.21	50.5	0.346	0.723	49	64	−15	114.35	610.12
Albania	76.87	11.27	7976.33	33.0	0.356	0.719	50	62	−12	103.76	707.67
Russian Federation	67.21	14.09	15258.16	43.7	0.429	0.719	51	42	9	227.03	1082.95
Kazakhstan	65.36	15.08	10234.32	30.9	0.408	0.714	52	50	2	156.57	678.65
Azerbaijan	70.85	13.03	8746.57	16.8	0.403	0.713	53	52	1	123.46	671.43
Bosnia and Herzegovina	75.50	13.01	8221.59	36.3	0.368	0.710	54	56	−2	108.90	631.96
Ukraine	68.62	14.65	6535.14	27.6	0.359	0.710	55	61	−6	95.24	446.13
Iran (Islamic Republic of)	71.91	14.02	11764.21	38.3	0.414	0.702	56	47	9	163.60	838.90
Macedonia	74.54	12.30	9486.86	42.8	0.364	0.701	57	59	−2	127.27	771.27
Brazil	72.93	13.78	10606.97	55.0	0.360	0.699	58	60	−2	145.45	769.99

(*Continued*)

TABLE A1. (Continued)

Country	Life expectancy at birth (years)	Expected years of schooling (years)	GNI per capita (PPP US$)	Gini (%)	Newer HDI	HDI 2010	Ranking HDI 2010	Ranking Newer HDI	Difference in rankings	Cost of an extra year in health (PPP US$)	Cost of an extra year in education (PPP US$)
Georgia	72.04	12.61	4901.91	40.8	0.295	0.698	59	79	−20	68.04	388.59
Venezuela	74.22	14.19	11846.23	43.4	0.409	0.696	60	48	12	159.62	834.99
Armenia	74.23	11.93	5494.61	30.2	0.321	0.695	61	72	−11	74.02	460.63
Ecuador	75.41	13.33	7931.24	54.4	0.328	0.695	62	66	−4	105.18	595.08
Belize	76.86	12.44	5693.06	59.6	0.278	0.694	63	83	−20	74.07	457.54
Colombia	73.41	13.30	8588.94	58.5	0.323	0.689	64	71	−7	117.00	645.66
Jamaica	72.33	11.70	7206.85	45.5	0.319	0.688	65	73	−8	99.64	615.72
Tunisia	74.33	14.50	7979.31	40.8	0.367	0.683	66	57	9	107.35	550.17
Jordan	73.11	13.11	5955.98	37.7	0.326	0.681	67	67	0	81.47	454.42
Turkey	72.23	11.84	13359.24	41.2	0.403	0.679	68	51	17	184.95	1128.23
Algeria	72.90	12.77	8320.16	35.3	0.365	0.677	69	58	11	114.13	651.32
Turkmenistan	65.33	12.97	7052.09	40.8	0.325	0.669	70	69	1	107.94	543.72
Dominican Republic	72.80	11.94	8272.56	48.4	0.330	0.663	71	65	6	113.64	693.07
China	73.47	11.38	7258.47	41.5	0.326	0.663	72	68	4	98.79	637.57
El Salvador	71.95	12.11	6498.11	46.9	0.308	0.659	73	76	−3	90.31	536.51
Sri Lanka	74.41	12.04	4886.32	41.1	0.293	0.658	74	80	−6	65.67	405.89
Thailand	69.30	13.47	8000.62	42.5	0.347	0.654	75	63	12	115.45	593.95
Gabon	61.33	12.66	12746.55	41.5	0.383	0.648	76	54	22	207.85	1006.98
Suriname	69.37	12.03	7092.90	52.8	0.301	0.646	77	78	−1	102.25	589.61
Bolivia	66.34	13.71	4357.24	57.2	0.255	0.643	78	88	−10	65.68	317.75
Paraguay	72.27	12.03	4585.32	53.2	0.263	0.640	79	87	−8	63.45	381.30
Philippines	72.34	11.54	4002.08	44.0	0.263	0.638	80	86	−6	55.32	346.87
Botswana	55.53	12.37	13204.19	61.0	0.325	0.633	81	70	11	237.79	1067.38
Moldova (Republic of)	68.90	12.04	3149.33	37.4	0.251	0.623	82	90	−8	45.71	261.66
Mongolia	67.28	13.47	3619.27	36.6	0.272	0.622	83	84	−1	53.79	268.68
Egypt	70.54	11.04	5889.20	32.1	0.312	0.620	84	74	10	83.48	533.58
Uzbekistan	68.18	11.52	3084.89	36.7	0.246	0.617	85	92	−7	45.24	267.70
Guyana	67.88	12.25	3302.06	43.2	0.248	0.611	86	91	−5	48.65	269.58

Namibia	62.07	11.84	6323.11	74.3	0.226	0.606	87	99	−12	101.86	534.11
Honduras	72.61	11.42	3750.11	55.3	0.238	0.604	88	95	−7	51.65	328.49
Maldives	72.32	12.45	5408.10	37.4	0.309	0.602	89	75	14	74.78	434.53
Indonesia	71.49	12.69	3956.84	37.6	0.279	0.600	90	82	8	55.35	311.82
Kyrgyzstan	68.42	12.58	2291.23	33.5	0.234	0.598	91	97	−6	33.49	182.11
South Africa	51.97	13.41	9812.13	57.8	0.304	0.597	92	77	15	188.80	731.85
Tajikistan	67.31	11.36	2019.88	33.6	0.215	0.580	93	100	−7	30.01	177.74
Viet Nam	74.91	10.38	2994.76	37.8	0.241	0.572	94	93	1	39.98	288.64
Morocco	71.84	10.49	4627.57	40.9	0.272	0.567	95	85	10	64.42	441.23
Nicaragua	73.77	10.77	2567.40	52.3	0.212	0.565	96	101	−5	34.80	238.34
Guatemala	70.77	10.64	4693.74	53.7	0.251	0.560	97	89	8	66.32	441.35
Cape Verde	71.94	11.22	3305.62	50.4	0.234	0.534	98	96	2	45.95	294.54
India	64.35	10.31	3337.37	36.8	0.239	0.519	99	94	5	51.86	323.80
Timor-Leste	62.06	11.18	5303.20	31.9	0.290	0.502	100	81	19	85.46	474.26
Swaziland	46.97	10.27	5132.03	50.7	0.228	0.498	101	98	3	109.26	499.55
Cambodia	62.17	9.84	1867.66	44.2	0.184	0.494	102	106	−4	30.04	189.78
Pakistan	67.16	6.79	2678.26	31.2	0.201	0.490	103	103	0	39.88	394.25
Congo	53.93	9.34	3257.64	47.3	0.203	0.489	104	102	2	60.40	348.91
Sao Tome and Principe	66.08	10.20	1917.63	50.6	0.184	0.488	105	105	0	29.02	188.02
Kenya	55.56	9.62	1627.74	47.7	0.164	0.470	106	113	−7	29.30	169.14
Bangladesh	66.94	8.14	1587.24	31.0	0.180	0.469	107	109	−2	23.71	195.02
Ghana	57.10	9.66	1385.47	42.8	0.162	0.467	108	115	−7	24.26	143.47
Cameroon	51.73	9.75	2196.89	44.6	0.181	0.460	109	107	2	42.47	225.26
Yemen	63.85	8.65	2386.63	37.7	0.200	0.439	110	104	6	37.38	276.01
Benin	62.31	9.22	1499.11	38.6	0.173	0.435	111	110	1	24.06	162.54
Madagascar	61.25	10.21	953.06	47.2	0.145	0.435	112	125	−13	15.56	93.34
Mauritania	57.28	8.06	2118.32	39.0	0.180	0.433	113	108	5	36.98	262.74
Papua New Guinea	61.59	5.23	2227.10	50.9	0.151	0.431	114	120	−6	36.16	425.86
Nepal	67.46	8.84	1200.79	47.3	0.154	0.428	115	119	−4	17.80	135.91
Togo	63.27	9.58	843.78	34.4	0.148	0.428	116	123	−7	13.34	88.04
Comoros	66.18	10.71	1176.07	64.3	0.142	0.428	117	127	−10	17.77	109.86
Lesotho	45.91	10.26	2021.15	52.5	0.164	0.427	118	114	4	44.02	196.98
Nigeria	48.42	8.86	2156.50	42.9	0.173	0.423	119	111	8	44.54	243.26

(*Continued*)

Table A1. (Continued)

Country	Life expectancy at birth (years)	Expected years of schooling (years)	GNI per capita (PPP US$)	Gini (%)	Newer HDI	HDI 2010	Ranking HDI 2010	Ranking Newer HDI	Difference in rankings	Cost of an extra year in health (PPP US$)	Cost of an extra year in education (PPP US$)
Uganda	54.14	10.42	1224.06	42.6	0.157	0.422	120	117	3	22.61	117.44
Senegal	56.23	7.52	1815.78	39.2	0.166	0.411	121	112	9	32.29	241.32
Haiti	61.68	6.84	949.00	59.5	0.116	0.404	122	136	−14	15.39	138.74
Angola	48.08	4.39	4941.20	58.6	0.161	0.403	123	116	7	102.78	1124.47
Djibouti	56.15	4.72	2471.38	39.9	0.156	0.402	124	118	6	44.01	523.46
Tanzania	56.95	5.33	1344.29	34.6	0.137	0.398	125	131	−6	23.61	252.06
Côte d'Ivoire	58.37	6.30	1624.86	48.4	0.144	0.397	126	126	0	27.84	257.90
Zambia	47.31	7.17	1358.52	50.7	0.130	0.395	127	133	−6	28.72	189.36
Gambia	56.59	8.60	1357.68	47.3	0.150	0.390	128	121	7	23.99	157.95
Rwanda	51.12	10.63	1190.34	46.7	0.150	0.385	129	122	7	23.29	111.97
Malawi	54.61	8.90	910.97	39.0	0.138	0.385	130	130	0	16.68	102.36
Guinea	58.86	8.62	953.46	43.3	0.139	0.340	131	129	2	16.20	110.65
Ethiopia	56.13	8.25	992.03	29.8	0.146	0.328	132	124	8	17.67	120.19
Sierra Leone	48.25	7.21	808.72	42.5	0.116	0.317	133	137	−4	16.76	112.24
Central African Republic	47.71	6.34	757.85	43.6	0.108	0.315	134	140	−6	15.89	119.62
Mali	49.23	7.95	1171.31	39.0	0.140	0.309	135	128	7	23.79	147.30
Burkina Faso	53.70	5.84	1214.83	39.6	0.131	0.305	136	132	4	22.62	208.19
Liberia	59.11	11.00	319.81	52.6	0.099	0.300	137	141	−4	5.41	29.08
Chad	49.22	5.99	1066.75	39.8	0.123	0.295	138	134	4	21.67	178.01
Guinea-Bissau	48.58	9.11	538.09	35.5	0.114	0.289	139	138	1	11.08	59.03
Mozambique	48.37	8.22	854.09	47.1	0.120	0.284	140	135	5	17.66	103.87
Burundi	51.37	9.57	401.57	33.3	0.109	0.282	141	139	2	7.82	41.97
Niger	52.49	4.31	675.38	43.9	0.094	0.261	142	142	0	12.87	156.85
Congo	48.01	7.76	291.23	44.4	0.084	0.239	143	143	0	6.07	37.52
Zimbabwe	47.00	9.24	176.17	50.1	0.072	0.140	144	144	0	3.75	19.06

Source: Human Development Report 2010: http://hdr.undp.org/en/media/HDR_2009_Tables_rev.xls

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Improving the Measurement of Human Development

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Abstract

We propose a new Human Development Index that involves a number of changes with respect to the present one, even though it keeps the basic structure of the index (namely, preserving "health", "education" and "material wellbeing" as the three basic dimensions of human development). The first change refers to the substitution of the arithmetic mean by the geometric mean, as a way of aggregating the different dimensions in a more sensible way. The second one leads to the introduction of distributive considerations in the evaluation of material wellbeing. The last change consists of the introduction of new variables to approach health and education, looking for a higher sensitivity of the index with respect to the differences between countries. These new variables are specially indicated for the analysis of human development in highly developed countries. Besides the conceptual discussion, that includes a characterization of the chosen aggregation formula, we present a comparative analysis of this new index and the standard one, focusing on the OECD countries.

Keywords: Human Development, multiplicative indices, distributive concerns, highly developed countries, HDI(2).

JEL classification: 015, 131

"A general goal for human development is to enhance the quality of human life. However, the concept "quality of human life" is not well defined. It is determined by a set of interrelated factors that cut across many disciplines with varied perspectives and paradigms. These include the prevailing culture, health status, economic performance, political and social conditions, the building of human capacity and capabilities, and institutional development…

However, these factors are not independent in their effects, nor do they act in harmony… (That speaks of) the complexity of measuring human development and achievement in the absence of a well-defined system of ranking."

Ismail Sirageldin,
Sustainable Human Development in the Twenty First Century

1. Introduction

Improving human welfare and fostering economic development are two basic goals of most democratic Governments. Evaluating the achievements of policy measures from that perspective requires having sensible indicators for those concepts. This is not an easy task, as it implies making a number of theoretical and practical compromises concerning the dimensions involved and the way of approaching them in terms of the available data.

There is a long tradition of taking the dynamics of the GDP as the key reference measure for economic growth, which in turn is supposed to approximate the level of economic development of a society. National statistical offices provide regular information on that variable which is systematically used to evaluate the overall performance of the country (growth rate, relative position with respect to other countries, etc.). The limits of this indicator are well known: the GDP only computes market transactions, it ignores qualitative or distributive aspects, it only provides a rough approximation of the cost of use of capital, it does not compute stocks of durables and infrastructures, etc. Yet we keep using this extremely simple indicator, partly because we are well aware of all those shortcomings and partly because it is positively correlated to several of the relevant aspects of economic development we would like to measure.

Using a single dimension to evaluate economic development appears as a weak methodological approach. A natural way of improving the analysis of economic development is, therefore, to build up multidimensional indicators that may account for several aspects related to human welfare and economic potential (environment, health, education, social integration, etc.). The construction of that type of indicators opens a whole line of research: which are the most relevant dimensions to be considered? How to approximate those dimensions by means of specific variables? How to aggregate those variables into a single indicator?

There has been a number of proposals in that direction, along different lines. Let us mention the United Nations 1954 report on the standards of living, the "basic needs approach" fostered by the International Labour Organization in 1974, the *Physical Quality of Life Index* (PQLI), due to Morris (1979) (reformulated by Ram (1982)), or that proposed by the Daj Hammarskjöld Foundation (Max-Neef (1984)). *Eurostat* has also set forth a protocol to approach sustainable development with a series of sensible indicators.[1] The need of multidimensional indicators for the assessment of economic development is already well established. The recent report by Stiglitz, Sen and Fitoussi (2009) is one of the last attempts to transform such a need into an institutional commitment, that should lead to a change in our national accounting systems. The construction of those indicators requires covering three differentiated but closely related stages. First, to build up a wide agreement on the relevant dimensions to be considered.

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[1] See http://epp.eurostat.ec.europa.eu/portal/page/portal/product_details/publication?p_product_code=KS-68-05-551

Second, to choose the variables that approximate those dimensions (variables that should be available on a regular basis through the existing statistical services). And third, to define the proper way of synthesizing those variables into an index that yields more operational the information involved and allows performing sensible comparisons.

The Human Development Index is probably the most successful multidimensional indicator nowadays. It was proposed by the United Nations in 1990 as a protocol to measure the countries' degree of development, based on Amartya Sen's idea of *functionings and capabilities* [see Sen (1985)]. This protocol identifies health, education, and material wellbeing as the key human functionings (first stage). The achievements in health, education and material wellbeing were associated with the variables *life expectancy at birth,* a mixture of *literacy rate* and *gross enrolment rate* (with weights of 2/3 and 1/3, respectively), and the *log of the standard per capita GDP*, respectively, suitably normalized (second stage). Finally, the *Human Development Index* (HDI, for short) consists of the *arithmetic mean* of the normalized values of those three variables (third stage).[2]

The HDI has been subject to a number of well-grounded criticisms, in spite of the improvement that it implies with respect to the mere comparison of per capita GDP values. The main criticisms refer to:[3]

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[2] The Human Development Index is complemented by other companion indices that focus on specific subjects, most notably gender and poverty. Two remarks are worth mentioning. One, that those indices use different types of mean in order to aggregate partial indicators. Two, that the poverty measure includes a specialized index for more developed countries. Both features are present in the proposal contained in this work.

[3] See the contributions in Anand & Sen (1994 a, b), Hicks (1997), Sagar & Najam (1999), Osberg & Sharpe (2002), Philipson & Soares (2001), Pinilla & Goerlich (2003), Foster, López-Calva & Székely (2003), Becker, Philipson & Soares (2005), Stiglitz, Sen & Fitoussi (2009), or Herrero, Soler & Villar (2010).

(a) *The number and nature of the selected dimensions*. There are some relevant aspects of human development that are missing, such as social integration or sustainability.

(b) *The choice of the variables that measure those dimensions*. Even though this is partly a practical matter (availability of data), it is not clear that the variables used to approximate health, education and material wellbeing are the most sensible ones.

(c) *The lack of concern for distributive issues*. It is only natural to think that the level of human development should compute not only "the size of the cake", but also the way in which it is distributed.

(d) *The nature of the three variables involved*. This feature makes it difficult to interpret the HDI as an average value (a summary statistic of a representative agent).

(e) *The additive structure of the index*. Aggregating the different components by the arithmetic mean has strong implications on their substitutability (linear indifference curves) and makes the index dependent on the normalization chosen for the different components.

(f) *The lack of theoretical justification of the formula*. This makes it difficult to analyze the suitability of this index vis a vis other alternatives. Moreover, it induces the use of the HDI as an ordinal measure (a criterion to produce a ranking) and not as a cardinal measure that would help evaluating the *size* of the differences between countries.[4]

Applied work has also pointed out the scarce sensitivity of the HDI when applied to developed countries. The reasons are clear. On the one hand, the type of variables chosen to approximate the three selected dimensions: life expectancy at birth tends to overweight the health component of those countries with a less dynamic demography; the index of education hardly reflects the existing human capital due to the excessive weight given to the literacy rate; and the use of logs flattens income differences. On the other hand, the aggregation formula: the arithmetic mean pays no attention to the dispersion of the values of the three components.

There are different ways to modify and improve the HDI and its companion indices, in order to provide a better picture of the degree of development of a society. Let us comment on some of them as a way of clarifying the nature of our proposal.

Adding new dimensions is clearly one of those ways of improvement. In particular dimensions related to the sustainability of the society, the availability of infrastructures, the presence of social conflicts, the degree of social integration, or the basic rights of the citizens, to name a few. This is an important venue that requires reaching a consensus not only on the additional dimensions to be considered, but also on the variables that measure them and the weights with which they should enter the final formula. Note also that some of the data needed to

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[4] This feature has raised some scepticism. Some researchers argue that the ranking produced by the HDI is not very different from that steaming from the per capita GDP, so that there is not a great need of such a multidimensional indicator (see for instance: Justin Wolfers, What does Human Development Measures? http://freakonomics.blogs.nytimes.com/2009/05/22/what-does-the-human-development-index-measure/).

implement an index of this sort are not provided by the standard statistical offices. Therefore, a structural change of this nature, that is certainly needed, calls for a sound and presumably long term work involving changes in the national accounting systems.

This is *not* the nature of the alternative formulation presented here. We rather focus on some *modifications of the present index* that try to make it a more suitable measure of human development. Our proposal stems from three different considerations. First, that changes in the way of measuring human development should not be too drastic in order to keep at least part of the achievements obtained so far in providing a measure that goes beyond the GDP (this is partly a cultural asset that is to be protected and partly an operational interest in keeping track of the former work). Second, that the additive structure of the index involves relevant shortcomings, as it makes the resulting ranking dependent on the normalization process. And third, that some of the deficiencies of the HDI mentioned above are specially relevant when we apply this methodology to highly developed countries, which is an incentive not to use them as a relevant source of information on a regular basis.

We therefore propose a new approach to define the human development index that may actually result in two different indicators, one of them specialized for highly developed countries. The improvements we propose refer to the following elements:

(i) The use of the geometric mean of the components, rather than the arithmetic mean, as a way of aggregating the three selected indicators, under a suitable theoretical justification.

(ii) The introduction of distributive considerations, as we believe distributional aspects are part of the basic features of the socio-economic performance.[5]

(iii)A change of the variables that measure the dimensions concerning health and education, bearing in mind the availability of data. We look for a better way of capturing the differences between developed countries.

We understand that the first two modifications are applicable to the HDI in general, whereas the change of variables proposed here is specially needed for the case of highly developed countries. The idea of designing an index specific for highly developed countries is conceptually a parallel exercise to that of the poverty index for some selected OECD countries, already set up by the United Nations. That is why we propose to call this special index the HDI(2).

This is not the first proposal that tries to provide a better approximation to the measurement of human development within developed countries. The *American Human Development Index*[6] is an alternative indicator designed to face some of the shortcoming already mentioned when dealing with highly developed countries. It keeps life expectancy at birth as the variable that measures health, modifies the education variable in order to give more weight to the upper level of studies (much in line with our approach), and substitutes the log of the per capita GDP by the

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[5] Taking care of distributional aspects may require a different definition of the variables that measure the achievements in health, education and material wellbeing. This is so because the standard way of introducing distributive considerations refers to the dispersion of the variable with respect to some average that can be regarded as the value of a representative agent [see however the proposal in Grim et al. (2008)]. The present way of measuring those variables, does not allow a clear interpretation of the index as some average value. As an illustration, consider the case of income. The use of logs in the income indicator makes it difficult to introduce distributive considerations. It is true that taking the log of the income permits one to interpret the resulting value as a welfare measure. Yet this principle is not applied to other variables and, from a descriptive viewpoint, hides again part of the existing differences between countries. This is especially arguable when we compare countries with a similar degree of development.

[6] See Burd-Sharps (2008), and http://www.measureofamerica.org/

log of the median income, applying their index to compare the States in the US. Herrero, Soler & Villar (2010), analyze the evolution of human development in Spain and its regions between 1980 and 2007. They provide, besides the standard analysis, an alternative HDI close to our proposal here.

The 2010 Human Development Report includes a new HDI, called here the HDI_{2010}. This new index preserves the three traditional dimensions (health, education, and material wellbeing) and the additive aggregation formula. It introduces some changes in the definition of the variables as well as in the normalization procedure. While life expectancy at birth is kept as the basic health indicator, the variables that approach education and material wellbeing are modified. The literacy rate and the gross enrolment rate are substituted by "mean schooling years" (for adults) and "school life expectancy" (for children), respectively. As for material wellbeing, the GDP is substituted by the Gross National Income (GNI) (besides using natural logs instead of base 10 logs). Concerning the normalization procedure, previous fixed caps are removed and substituted by actual maximum and minimum values for each variable.

The rest of the paper goes as follows. Section 2 is devoted to the analysis of the alternative aggregation formula, the geometric mean, including an easy characterization that conveys theoretical support to this formula and permits its use in a cardinal sense. Section 3 introduces a new measure of material wellbeing that takes into account explicitly the income distribution (the inequality adjusted per capita GDP). Section 4 presents our proposals to approximate health *(life potential)* and education *(expected years of schooling over compulsory education)*. Those variables are specifically designed for highly developed countries. Sections 3 and 4 include an empirical illustration of the impact of the proposed changes in 26 OECD countries with data of

2007.[7] Section 5 further extends the empirical analysis and provides a comparison of the results derived of using the HDI(2) with respect to the traditional HDI for the selected OECD countries. The comparison of our proposal and the HDI_{2010} is the subject of section 6. Section 7 gathers some final comments and recommendations.

2. The new aggregation formula

2.1. The additive structure of the HDI

The use of the arithmetic mean as a way of aggregating the three partial indices chosen to approximate health, education and material wellbeing, is a notorious source of discomfort. On the one hand, the additive structure of the index implies a very peculiar trade-off between the different components. More specifically, such a structure amounts to assuming full substitutability between all components (linear indifference curves). On the other hand, the index generates a ranking that is sensitive to the normalization of the different variables (this is so because changing the normalization amounts to modifying the weights with which those variables enter the index).

From a different perspective, both shortcomings (the lack of theoretical justification of

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[7] We have excluded some countries for missing data (México and Turkey) and also Luxembourg for a different reason (it is a very small country with an extremely high per capita GDP).

this formula ad the sensitivity to the normalization) make it difficult to use it as a cardinal measure, a point always made clear by the Human Development Report. Yet having an idea of the relative distances between countries would clearly improve the informative content of the index.

We propose a twofold modification of the aggregation procedure for the HDI. On the one hand, normalizing the variables that measure health, education and material wellbeing in terms of the percentage of a maximum value (which amounts to setting all min values equal to zero). On the other hand, and this is the major change, substituting the arithmetic mean of the normalized variables by the geometric mean. The geometric mean can be characterized in terms of reasonable axioms (e.g. Herrero, Martínez & Villar (2010)) and exhibits much better properties than the arithmetic mean in this context.

We obtain in this way an index that solves the drawbacks mentioned above as it produces a ranking that does not depend on the choice of units of the different dimensions and, moreover, exhibits a decreasing rate of substitution between the variables, as the standard theory suggests.

Observe that the geometric mean is nothing else than the generalized mean of order 0, whereas the arithmetic mean is that of order 1. This is in line with the construction of other indicators of human development that use generalized mean of order -1 (the harmonic mean, to measure gender discrimination) or that of order -3 (to measure poverty).[8] The geometric mean (and all generalized means of a smaller order) penalizes the differences in the values of the constituent variables. That is, this type of index takes into account negatively the dispersion of

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[8] There are several contributions that suggest the use of generalized means in this context, including alternative characterizations. See Foster, López-Calva & Székely (2005), Seth (2009), (2010), Villar (2009).

the partial indices that are being aggregated.[9] This implies that, in order to have a high position in the ranking produced by this indicator, one has to have high marks in all constituent variables and not only in some of them.

We present in section 2.2 a formal characterization of the geometric mean in terms of simple axioms. We show that this aggregation formula can be characterized by means of three properties: *Neutrality* (all normalized characteristics are equally important), *Scale* (if all variables are equal then the index takes on that very same value), and *Ratio Consistency* (a common change in the value of a variable keeps constant the ratio of the initial values).

Assuming those properties amounts, therefore, to evaluate a vector of three components (α, β, γ) referring to normalized values of health, education and material wellbeing, as follows:

$$I(\alpha, \beta, \gamma) = (\alpha)^{1/3} (\beta)^{1/3} (\gamma)^{1/3}$$

The axiomatic support of the aggregation formula not only suppresses the arbitrary nature of the index but also conveys a cardinal dimension. As a consequence, we can perform quantitative comparisons and not only to generate and ordinal ranking.

2.2. A pinch of theory: Axiomatization of the new index

There are different ways of choosing an aggregation formula, in order to synthesize several indicators into a single number. One may recur to persuasion or invoke tradition to

[9] See the discussion in Seth (2009), (2010).

defend an intuitive and sensible aggregation function. For instance, one may think of the arithmetic mean, the geometric mean, the harmonic mean, or any other generalized mean. They are standard aggregators, simple to compute, well known, and are widely used (also in the Human Development Reports). The problem is, of course, how to choose among them, why one and not the other. This requires a comparative analysis of the implications of their use prior to selecting the formula that fits better our purposes.

An alternative way, with a long tradition in economics, is to choose solutions to economic problems on the basis of the normative and/or operational properties of those solutions. This is the so-called *axiomatic method*[10] that aims at identifying a solution function with a unique set of intuitive properties or *axioms*. In that case, choosing a given set of axioms turns out to be equivalent to choosing a given formula. We apply here this methodological approach in order to identify the geometric mean as the only way to aggregate the achievements in health, education and well-being that satisfies three intuitive requirements: *neutrality, scale,* and *ratio consistency.*

Let us formalize these ideas.

Suppose that we want to define an evaluation index for a given society that aggregates three (normalized) variables.[11] We define a **social state** as a vector (α, β, γ) with *3* components, each of which belongs to the interval [0,1] (that is, the values those characteristics are already normalized so that the differences in their mean values have been cancelled). Therefore, $\Omega = [0,1]^3$ is the space of admissible social states.

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[10] See Thomson (2001) for a wide exposition of the advantages and disadvantages of the axiomatic method.

[11] Let us remark that the argument below can be extended to any arbitrary (finite) number of components (at the cost of making the proof much more cumbersome). For a more general approach see Herrero, Martínez & Villar (2010) or Villar (2009).

A **Social Evaluation Index** is a continuous single-valued mapping $I:\Omega \to \mathbf{R}$ that provides a numerical evaluation of social states.[12]

We first introduce two basic requirements on the social evaluation index: neutrality and scale. *Neutrality* makes it explicit that all characteristics enter the evaluation function on an equal foot. That can be formalized by requiring that a permutation of the characteristics does not affect the social evaluation (recall that all variables vary in the interval [0, 1], so that the differences in the units of measurement have already been neutralized). *Scale* fixes the value of the index when the social state is uniform (i.e. all entries are identical), by choosing precisely that very same value. Formally:

- **Neutrality**. For each point $(\alpha,\beta,\gamma)\in\Omega$, if $\pi(\alpha,\beta,\gamma)$ denotes a permutation of its elements, then: $I\left[\pi(\alpha,\beta,\gamma)\right]=I(\alpha,\beta,\gamma)$.

- **Scale**. Let $p\in[0,1]$. Then, $I(p,p,p)=p$.

The last property, *Ratio Consistency*, requires that the relative value of the indices of two social states with a common component does not depend on the value of that common component. Formally:

- **Ratio Consistency.** Let $(\alpha,\beta,\gamma),(\alpha,\beta',\gamma')\in\Omega$ be two strictly positive social state vectors, with the same first component α. If that common component

[12] Note that we introduce the requirement of continuity in the very definition of the index. That is, we focus our discussion on those mappings for which small changes in the variables imply small changes in the index.

changes to a different one, α', the ratio of the associated indices does not change. That is,

$$\frac{I(\alpha,\beta,\gamma)}{I(\alpha,\beta',\gamma')} = \frac{I(\alpha',\beta,\gamma)}{I(\alpha',\beta',\gamma')}$$

This property says the following. Suppose that countries A and B have the same values concerning the health variable and different values with respect to education and material wellbeing, all positive, so that the overall index of country A is twice that of country B. Now both countries experience an improvement in health that changes the corresponding variable by exactly the same amount. That change obviously alters the associated development indices. Ratio Consistency implies that the new index of country A is still twice the new index of country B. That is to say, the relative value of the index is not affected by an equal change of a common value of a given variable.

Remark.- *We ask this property to hold just for one component (the first one, in our definition) for the sake of parsimony. When combined with "neutrality" this property actually applies to any common value.*

Note that this consistency requirement is cardinal in nature and involves a separability feature in the evaluation index.

The following result is obtained:

Theorem: *A social evaluation index I(.) satisfies neutrality, scale and ratio consistency, if an only if it takes the form:*

$$I(\alpha,\beta,\gamma)=\alpha^{1/3}\beta^{1/3}\gamma^{1/3}$$

Moreover, those properties are independent.

<u>*Proof.-*</u>

First note that scale implies $I(\alpha,\alpha,\alpha)=\alpha$, for all $\alpha\in[0,1]$. By neutrality, the property of ratio consistency can be applied to any component of vector (α,β,γ).

Take now a vector $(\alpha,\beta,\gamma)>>0$. By ratio consistency and neutrality we can write:

$$\frac{I(\alpha,\alpha,\alpha)}{I(\alpha,\alpha,\beta)}=\frac{I(\alpha,\alpha,\alpha)}{I(\beta,\alpha,\alpha)}=\frac{I(\alpha,\beta,\gamma)}{I(\beta,\beta,\gamma)}$$

And thus,

$$I(\alpha,\alpha,\beta)=\alpha\frac{I(\beta,\beta,\gamma)}{I(\alpha,\beta,\gamma)}$$

In a similar way, $\frac{I(\beta,\beta,\beta)}{I(\beta,\beta,\gamma)}=\frac{I(\alpha,\beta,\gamma)}{I(\alpha,\gamma,\gamma)}$, so that $I(\beta,\beta,\gamma)=\beta\frac{I(\gamma,\gamma,\alpha)}{I(\alpha,\beta,\gamma)}$, and $I(\alpha,\alpha,\beta)=\alpha\beta\frac{I(\gamma,\gamma,\alpha)}{[I(\alpha,\beta,\gamma)]^2}$

Finally, $I(\gamma,\gamma,\beta)=\gamma\frac{I(\alpha,\alpha,\beta)}{I(\alpha,\beta,\gamma)}$. Substituting, $I(\alpha,\alpha,\beta)=\alpha\beta\gamma\frac{I(\alpha,\alpha,\beta)}{[I(\alpha,\beta,\gamma)]^3}$.

Therefore, we conclude:

$$I(\alpha,\beta,\gamma)=\alpha^{1/3}\beta^{1/3}\gamma^{1/3}$$

(ii) To separate the properties let us consider the following indices:

(1) $I(\alpha,\beta,\gamma)=\frac{1}{3}(\alpha+\beta+\gamma)$. It satisfies neutrality, scale but not ratio consistency. (2) $I(\alpha,\beta,\gamma)=\frac{\alpha\beta\gamma}{3}$. It satisfies neutrality, ratio consistency and not scale.

(3) $I(\alpha,\beta,\gamma)=\alpha^{a}\beta^{b}\gamma^{c}$, with $a+b+c=1$ and not all of them are equal. It satisfies ratio consistency and scale but not neutrality. **Q.e.d.**

This theorem says that, among those indices that satisfy neutrality and scale, ratio consistency determines that the evaluation formula is given by the geometric mean of the corresponding normalized values of the chosen variables. Note that this aggregator is a special case of the family of the generalized means of order $q \in \mathbf{R}$, $\mu_q=\left[\frac{1}{k}\sum_{j\in K}x_j^q\right]^{\frac{1}{q}}$, for $q=0$.

It is worth noting that if we change the units of any dimension in the normalization of its component, not only the ranking of the countries is preserved, but also the relative size of the indices. Thus, the new HDI allows for ordinal and cardinal comparisons among the countries.

Remark.- *This Theorem differs from the characterization in Herrero, Martínez & Villar (2010) in two respects. One, that it refers to normalized values at the population level, rather than individual values (there a social state is a matrix that describes the distribution of each* *characteristic within the population). And two, that it uses a stronger separability property (ratio consistency) but it requires neither "minimal lower boundedness" nor "monotonicity".*[13]

3. The concern for equality: Inequality-adjusted income

The lack of concern for distributive issues in the income dimension of human welfare is perhaps one of the most surprising features of the HDI. There are statistics that approximate inequality for most of the countries and we have a well-established theory that permits one to link the evaluation of the size and distribution of income simultaneously. Moreover, there are already a number of contributions that suggest ways of introducing equality concerns in this particular context.[14]

Material wellbeing is to be measured by the per capita GDP suitably adjusted by the income distribution. As in the UN traditional methodology, we assume that the standard per capita GDP, expressed in terms of PPP US dollars, is the basic variable. We propose, however, to suppress the use of logs in order to fully capture the differences among countries in that aspect. Moreover, and this is also a relevant part of the proposal, we deflate that figure by the corresponding inequality index.

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[13] Minimal lower boundedness says that if a column is zero, then the index is zero. This property is obviously implied by the combination of our axioms. Monotonicity, however, is neither implied nor required, which gives us more flexibility in the definition of the egalitarian equivalent value.

[14] See, for instance, Anand & Sen (1994b), Hicks (1997), Foster, López-Calva & Székely (2005), Herrero, C., Martínez, R. & Villar, A. (2010), Seth (2009), (2010), Villar (2009).

The standard way of conveying a normative content to an inequality measure is that of interpreting inequality as a welfare loss, in the tradition of Dalton, Atkinson, Sen and Kolm, to name a few representative thinkers. To do so let $\mathbf{y} = (y_1, y_2, ..., y_n)$ denote the income distribution of a society and let $W(\mathbf{y})$ a social welfare measure of that distribution. Then, define the ***egalitarian equivalent income***, y^e, as that amount of income that equally distributed would yield the same social welfare than the current income distribution. That is, y^e is the value that satisfies the following equation:

$$W(y_1, y_2, ..., y_n) = W(y^e, y^e, ..., y^e)$$

This value y^e always exists, provided W is a continuous function defined on a compact domain. Moreover, under reasonable hypothesis (quasi-concavity) the egalitarian equivalent income is always below per capita income. We can, therefore, define an inequality measure as follows:

$$I(\mathbf{y}) = 1 - \frac{y^e}{\mu(\mathbf{y})}$$

where $\mu(\mathbf{y})$ is the mean income. This formula tells us that inequality can be understood as the welfare loss due to the difference between the egalitarian equivalent income and the mean value.

This can be rewritten as follows:

$$y^e = \mu(\mathbf{y})\left[1 - I(\mathbf{y})\right]$$

This is the type of indicator we propose for the measurement of material wellbeing. The choice of the right inequality index can be done making use of the properties we deem relevant (e.g. decomposability, degree of preference for equality, etc.).[15]

For the sake of the empirical application presented below, we propose the use of the Gini coefficient as a sensible way of measuring inequality. That is, we shall measure material wellbeing in terms of the following inequality adjusted income:

$$IAI_G = GDPpc\left[1 - G\right]$$

$GDPpc$ is the per capita Gross Domestic Product (as an approximation of the mean income) and G the Gini coefficient that measures income dispersion. By so doing, we deflate the GDPpc by inequality, measured with the Gini coefficient. Therefore, if we find two societies with identical GDPpc, we consider more developed the one which is more egalitarian.

The Gini coefficient has some well-known shortcomings, such as the lack of additive decomposability or the insensitivity to the size of the income differences (a property sometimes called "homothetic distributivity"). And it has also many advantages, derived from the multiple ways of writing and interpreting this index, easily derived from the Lorenz curve. Be as it may, it is the most frequently used inequality measure in empirical work and therefore has become an index supplied regularly by most statistical offices.

The data required to construct this variable can be obtained from United Nations (Human Development Report 2009)[16] and the OECD website.[17]

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[15] See, for instance, Cowell (1995), Sen & Foster (1997), Goerlich & Villar (2009).

Table 1: The Gini Coefficient, the per capita GDP and the Inequality Adjusted Income in OECD countries (2007)

	Gini	pcGDP	Ranking pcGDP	IAI(G)	Ranking IAI(G)	Difference
Norway	0.28	53433	1	38471.76	1	0
Ireland	0.33	44613	3	29890.71	2	1
Switzerland	0.28	40658	4	29273.76	3	1
Sweden	0.23	36712	7	28268.24	4	3
United States	0.38	45592	2	28267.04	5	-3
Netherlands	0.27	38694	5	28246.62	6	-1
Denmark	0.23	36130	8	27820.1	7	1
Austria	0.27	37370	6	27280.1	8	-2
Iceland	0.28	35742	10	25734.24	9	1
Belgium	0.27	34935	12	25502.55	10	2
Finland	0.27	34526	14	25203.98	11	3
Australia	0.3	34923	13	24446.1	12	1
Canada	0.32	34812	9	24352.16	13	-4
France	0.28	33674	16	24245.28	14	2
Germany	0.3	34401	15	24080.7	15	0
U. Kingdom	0.34	35130	11	23185.8	16	-5
Japan	0.32	33632	17	22869.76	17	0
Spain	0.32	31560	18	21460.8	18	0
Italy	0.35	30353	19	19729.45	19	0
Greece	0.32	28517	20	19391.56	20	0
New Zealand	0.34	27336	21	18041.76	21	0
Czech Rep.	0.27	24144	22	17625.12	22	0
Slovak Rep.	0.27	20076	24	14655.48	23	1
Portugal	0.38	22765	23	14114.3	24	-1
Hungary	0.29	18755	25	13316.05	25	0
Poland	0.37	15987	26	10071.81	26	0
Average	0.31	33248.85		23290.20		
Coef. Variation	0.1307	0.2485		0.2613		

[16] Data on per capita GDP, expressed in terms of PPP 2005 dollars.

[17] Dataset: Income distribution, Inequality, Income and population measures, Gini coefficient after taxes.

Table 1 provides the basic data for most OECD countries. Introducing the inequality deflator increases the coefficient of variation in some 5 % and produces small changes in the ranking.[18] Note, however, that the UN index of material wellbeing involves taking logs of that variable, which produces an enormous reduction in the variability (as we shall see later, the coefficient of variation of that index in the HDI is around eight times smaller than the per capita GDP).

4. The new variables (specially fit for highly developed countries)

We propose here a new set of variables that allows for a more accurate approximation of the health and education dimensions and are intended to improve the sensitivity of those partial indicators (most specially in highly developed countries). Besides, we aim at using a set of variables that permits one to interpret the resulting HDI as an average (the value of a representative agent of the society under consideration).[19]

Needless to say, there are several alternative ways of modifying the existing variables in

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[18] Let us recall that the coefficient of variation is a dispersion measure consisting of the ratio between the standard deviation and the average, that is unit-free.

[19] Note that the standard HDI is a composition of three variables that are very different in nature. The per capita GDP (without logs!) can be interpreted in terms of the expected value of an individual picked at random in this society. Life expectancy at birth may be interpreted this way only with respect to the newborn, but it tells very little about the whole population. The combination of the literacy index and the gross enrolment rates generates a variable of still a different nature that cannot be nailed down to any sensible expected value. So the aggregation of those three variables cannot be interpreted in terms of a representative individual.

order to achieve those goals. What the best alternative indicator is depends on data availability, always a limiting factor, and on the "domain" of application, that is, the universe of societies on which we want to apply the indicator. Bearing in mind the restriction on data availability, we focus here on the domain consisting of highly developed countries. We believe that a multidimensional index of this nature requires some adjustments depending on the level of development of different groups of countries (as it is already acknowledged when analyzing poverty measures and also in the presentation of the HDI figures). This is important because the adherence to new measurement standards requires those new indicators to provide a better description of the reality they refer to. This is not the case so far for highly developed countries with the present HDI, for reasons already discussed.

4.1. Health: Life potential

Life expectancy (at birth) is a variable constructed in such a way that it turns out to be independent on the demographic structure. Besides, it tends to over-weight the health component of those countries with a higher share of old people. This last aspect is most arguable in the context of evaluating development capabilities for it ignores the differences in the present and future working age population.

Most developed countries exhibit very high values of life expectancy at birth, with a small variance, while they exhibit more relevant differences in the demographic structure (in particular in the share of young people in the population, that in some cases is linked to the arrival of new immigrants in late years). We believe that those differences are actually more important in developed countries than those corresponding to life expectancy at birth, when we come to assess human development possibilities. We therefore propose to substitute this variable for that of "life potential".

Life Potential measures the life expectancy of a representative individual in the population (Goerlich & Pinilla (2005)). To define this variable we first consider the number of years that individuals of age x are expected to live at time t (typically the present) and aggregate them:

$$B = \sum_{x=0}^{\infty} N_x e_x$$

Here, N_x is the number of people of age x and e_x is the expected number of years that people of that age will live. Life potential obtains from taking the per capita value of this variable. That is,

$$b = \frac{1}{N} \sum_{x=0}^{\infty} N_x e_x \ ,$$

N being the population size. This variable provides a measure of the average life expectancy of the population, taking into account its demographic structure.

The data required to calculate the life potential index can be obtained from the Human Mortality Database that provides both the life tables and the distribution of the population by age for almost all the countries in the OECD.[20]

One may argue that life potential is still further away from approaching the health condition of a population than life expectancy. True as that may be, we understand that the

[20] See http://www.mortality.org/ . See also the Health Database of the OECD, or the Life Tables of the World Health.

capability approach that informs the construction of human development indices is better served by this variable which incorporates an indirect estimate of potential economic growth in terms of the structure of the labour force.

Table 2: Life Expectancy and Life Potential per capita in OECD countries (2007)

	LE (years)	Ranking LE (RLE)	LP (years)	Ranking LP (RLP)	Difference (RLE-RLP)
Australia	81.4	4	45.96	2	2
Austria	79.9	12	41.68	14	-2
Belgium	79.48	16	41.47	15	1
Canada	80.62	9	44.31	5	4
Czech Republic	76.36	23	39.25	25	-2
Denmark	78.22	22	40.98	19	3
Finland	79.48	17	41.35	17	0
France	80.98	6	43.82	7	-1
Germany	79.78	14	39.81	23	-9
Greece	79.12	19	40.15	21	-2
Hungary	73.3	26	36.80	26	0
Iceland	81.76	2	46.75	1	1
Ireland	79.66	15	45.92	3	12
Italy	81.1	5	41.14	18	-13
Japan	82.66	1	41.38	16	-15
Netherlands	79.84	13	42.79	10	3
New Zealand	80.14	11	45.68	4	7
Norway	80.5	10	43.69	8	2
Poland	75.52	24	40.10	22	2
Portugal	78.58	21	40.87	20	1
Slovak Republic	74.62	25	39.64	24	1
Spain	80.74	8	42.62	11	-3
Sweden	80.8	7	42.20	13	-6
Switzerland	81.7	3	43.09	9	-6
U. Kingdom	79.36	18	42.32	12	6
United States	79.12	20	44.02	6	14
Average	79.4131		42.2230		
Coef. Of variation	0.0276		0.0550		

Table 2 compares the data on life expectancy at birth and life potential in 26 selected OECD countries. The picture we get with one or the other variable is rather different. To start with, the coefficient of variation of life potential is twice that of life expectancy (i.e. life potential discriminates much more than life expectancy). The rakings produced by both measures are rather different as well. The last column of the table tells us about the changes in the ranking, to be interpreted as the number of positions that a country advances in the ranking of life potential with respect to that of life expectancy. Note that Japan and Italy lose 15 and 13 positions, respectively, whereas the United States advances 14 and Ireland 12.

4.2. Education: Expected years of schooling

The index of education used in the HDI does not reflect the differences in human capital in developed countries. That is mostly due to the excessive weight given to the literacy rate. In countries with a well-established compulsory education system (that in many cases involves more than 9 years of school attendance), the present index of education hides most of the relevant differences. It is basically non-compulsory education what makes the difference, as it reflects the society's investment in human capital beyond what is legally required. Moreover, this variable exhibits much larger differences between developed countries than that of compulsory education, which is rather uniform.

There are several ways of giving more weight to non-compulsory education. One is to add up all students that reach some reference level, as this implies that those with higher level of studies are computed several times (that is the approach followed by the American Human Development Index). An alternative way is computing the average number of years of school attendance or the percentage of working age population with non-compulsory studies (as in Herrero, Soler & Villar (2010)). Here we propose to use the variable "expected years of education between 15 and 29" used by the OECD when assessing education and the labour market.[21] This variable has several advantages: it is already used as an international standard, it captures precisely the extent of non-compulsory education,[22] and it has the nature of an average.

The expected years of education at 15, in summary, is a variable that permits to capture much better the differences in human capital among developed countries. Let us mention that, on average, a person living in an OECD country who is 15 years-old in 2007 can expect to remain in school for an additional 6.9 years. Yet the differences range from four years (e.g. Turkey) to more than eight (e.g. Finland, Iceland).

The data on that variable are available from the OECD.[23]

Table 3 below shows the differences between both forms of measuring the educational achievements. We present the data corresponding to the "gross enrolment rate" and the "UN index of education" to illustrate the effect of the type of index chosen (i.e. the weight of the literacy rate on the overall education measure). Differences are outstanding: the coefficient of variation of the expected years of schooling (EYS) is 50 % higher than that of the gross enrolment rate and six times higher than that corresponding to the index of education. The last column gives us, as in the former table, the number of positions that a country gains or loses

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[21] See http://www.oecd.org/dataoecd/41/25/43636332.pdf

[22] Compulsory education in most developed countries ends at 15 years.

[23] See http://stats.oecd.org/index.aspx, http://dx.doi.org/10.1787/664770480457.

when using the variable expected years of schooling rather than the gross enrolment rate.[24] Differences are simply outstanding: nine countries out of 26 move more than ten positions upwards or downwards!

Table 3: Expected Years of Schooling (EYS), Gross Enrolment Rates (GER) and UN Education Index (EI) in OECD countries (2007)

	GER	EI	Ranking GER	EYS	Ranking EYS	Difference
Australia	114.2	0.993	1	6.8	10	-9
Austria	90.5	0.962	17	6.5	18	-1
Belgium	94.3	0.974	13	6.8	11	2
Canada	99.3	0.991	6	6.5	19	-13
Czech Republic	83.4	0.938	24	6.7	14	10
Denmark	101.3	0.993	5	7.8	6	-1
Finland	101.4	0.993	4	8.5	1	3
France	95.4	0.978	12	7.6	7	5
Germany	88.1	0.954	21	7.9	5	16
Greece	101.6	0.981	3	6.4	20	-17
Hungary	90.2	0.96	18	7.3	9	9
Iceland	96	0.98	11	8.5	2	9
Ireland	97.6	0.985	8	5	26	-18
Italy	91.8	0.965	16	6.7	15	1
Japan	86.6	0.949	23	5.8	24	-1
Netherlands	97.5	0.985	9	8	3	6
New Zealand	107.5	0.993	2	6.8	12	-10
Norway	98.6	0.989	7	6.7	16	-9
Poland	87.7	0.952	22	8	4	18
Portugal	88.8	0.929	20	5.9	23	-3
Slovak Republic	80.5	0.928	26	6.1	21	5
Spain	96.5	0.975	10	5.4	25	-15
Sweden	94.3	0.974	14	7.5	8	6
Switzerland	82.7	0.936	25	6.8	13	12
United Kingdom	89.2	0.957	19	6	22	-3
United States	92.4	0.968	15	6.7	17	-2
Average	94.1308	0.9685		6.8731		
Coef. of variation	0.08010	0.0209		0.1296		
Source: OECD						

[24] We think it more sensible the comparison of the EYS variable with the GER variable as the index of education involves an elaboration that distorts the meaning of the comparison.

5. The HDI(2) vis a vis the classic HDI in the OECD

In former sections we have introduced and discussed the alternative variables we propose to measure the achievements in health, education and material wellbeing. We have also provided a comparison between those new variables and the ones traditionally used by UN.

We call HDI(2) to the index that uses all the new variables and the new aggregation formula.

We present here an empirical illustration on the ways in which the HDI(2) measures human development, relative to the standard HDI, with respect to the 26 selected OECD countries (data corresponding to 2007). The comparative analysis is made not only with respect to the final indices, but also with respect to their three different components in order to underline the effects of the normalization choices. We analyze the changes in the values as well as the changes in the ranking.

Following the approach of the construction of the HDI, we first normalize the partial indices so that all they range within the interval [0,1]. In order to do so we choose a high enough value for each variable and express each measure as the fraction of that reference value (i.e. we simply divide the actual value by the reference one, which amounts to set the min value of each variable equal to zero). In this way our indicator turns out to be robust with respect to the choice of the reference values: both the ranking obtained and the relative values of any pair of countries are independent on the normalization chosen. We have, therefore, a measure that allows us to perform cardinal comparisons besides comparing the ranking of the countries. Be as it may, we select normalization values that seem reasonable for each variable, in the sense that they could be kept for some years to come, even if societies progress substantively.

The normalization values proposed are the following: 50 years for the life potential, 10 years for the expected years of schooling, and 60,000 PPP 2005 $ deflated by the OECD average inequality rate as measured by the Gini coefficient. That is, we define:

$$lp = \frac{LP}{50}, \quad eys = \frac{EYS}{10}, \quad iai_G = \frac{IAI_G}{60,000(1-0.31)}$$

The precise formula for the HDI(2) is therefore:

$$HDI(2) = (lp)^{1/3}(eys)^{1/3}(iai_G)^{1/3}$$

The components and values of the HDI and the HDI(2) are presented in Tables 4 and 5 below. They show that the HDI(2) is much more sensitive to the differences between countries than the HDI, which results in a much larger variability. Measuring the variability of the data in terms of the coefficient of variation, yields a clear cut outcome: the HDI(2) has a dispersion three times that of the HDI. Looking at the different components we observe that the change in the dispersion of the health indicator is very small (an increase of 15 %), whereas that of education is much larger (the coefficient of variation of the “expected years of schooling” is three times that of the UN index of education). The largest increase variability corresponds, not surprisingly, to the material wellbeing component, mostly as a result of removing the log transformation. Table 6 below, obtained from the data in Tables 4 and 5, summarizes those outcomes.

Table 4. The HDI and its components. International comparison. 2007				
	Life expectancy index	Education index	GDP index	HDI
Norway	0.925	0.989	1.000	0.971
Australia	0.940	0.993	0.977	0.970
Iceland	0.946	0.980	0.981	0.969
Canada	0.927	0.991	0.982	0.966
Ireland	0.911	0.985	1.000	0.965
Netherlands	0.914	0.985	0.994	0.964
Sweden	0.930	0.974	0.986	0.963
France	0.933	0.978	0.971	0.961
Japan	0.961	0.949	0.971	0.960
Switzerland	0.945	0.936	1.000	0.960
Finland	0.908	0.993	0.975	0.959
United States	0.902	0.968	1.000	0.956
Austria	0.915	0.962	0.989	0.955
Denmark	0.887	0.993	0.983	0.955
Spain	0.929	0.975	0.960	0.955
Belgium	0.908	0.974	0.977	0.953
Italy	0.935	0.965	0.954	0.951
New Zealand	0.919	0.993	0.936	0.950
Germany	0.913	0.954	0.975	0.947
United Kingdom	0.906	0.957	0.978	0.947
Greece	0.902	0.981	0.944	0.942
Portugal	0.893	0.929	0.906	0.909

Czech Republic	0.856	0.938	0.916	0.903
Poland	0.842	0.952	0.847	0.880
Slovak Republic	0.827	0.928	0.885	0.880
Hungary	0.805	0.960	0.874	0.879
Source: UN				

Table 5. The HDI(2) and its components. International comparison. 2007

	Life potential index	Expected years of education index	Inequality adjusted income index	HDI(2)
Norway	0.8738	0.67	0.9293	0.816
Iceland	0.9350	0.85	0.6216	0.791
Netherlands	0.8557	0.80	0.6823	0.776
Sweden	0.8441	0.75	0.6828	0.756
Denmark	0.8196	0.78	0.6720	0.755
Finland	0.8271	0.85	0.6088	0.754
Switzerland	0.8619	0.68	0.7071	0.746
United States	0.8805	0.67	0.6828	0.739
France	0.8765	0.76	0.5856	0.731
Australia	0.9193	0.68	0.5905	0.717
Germany	0.7962	0.79	0.5817	0.715
Austria	0.8336	0.65	0.6589	0.709
Belgium	0.8293	0.68	0.6160	0.703
Canada	0.8863	0.65	0.5882	0.697

Ireland	0.9183	0.50	0.7220	0.692
United Kingdom	0.8463	0.60	0.5600	0.658
New Zealand	0.9136	0.68	0.4358	0.647
Japan	0.8276	0.58	0.5524	0.642
Italy	0.8229	0.67	0.4766	0.640
Greece	0.8030	0.64	0.4684	0.622
Spain	0.8524	0.54	0.5184	0.620
Czech Republic	0.7849	0.67	0.4257	0.607
Hungary	0.7359	0.73	0.3216	0.557
Slovak Republic	0.7928	0.61	0.3540	0.555
Portugal	0.8175	0.59	0.3409	0.548
Poland	0.8019	0.80	0.2433	0.538
Source: OECD, UN, Human Mortality Database and Eurostat.				

Table 6: Coefficients of Variation (HDI, HDI(2) and its components, 2007)				
	Health	**Education**	**Income**	**Global Index**
United Nations' variables	0.0479	0.0415	0.0339	0.0381
New Variables	0.0550	0.1296	0.2613	0.1139
% of the new variables w.r.t. United Nations' variables	115	312	772	299

One may argue against that type of comparison because the HDI only intends to provide an ordinal measure of human development in order to generate a ranking (computing the coefficient of variation would not be meaningful in that case). So let us consider the *changes in the ranking* which are derived from our approach to measuring human development in highly developed countries. Table 7 below gives us that information. A mere ocular inspection tells us that the HDI(2) is really a different way of approaching human development. There are many and large changes in the global index (up to ten positions out of 26). The analysis of partial indices is quite informative, as already pointed out. Note that most of the changes in the ranking occur within the Education component, followed by the Health component. Material wellbeing does not change very much and those changes are obviously due to the differences in inequality (see Table 3 above).[25] This is interesting because, in spite of the large differences in the coefficient of variation with respect to the income dimension, the induced changes in the ranking are small. The opposite happens for the health dimension.

Table 7.[26] Ranking differences between the HDI(2) and the HDI, and its components. 2007

	Health	Education	Material wellbeing	Human development
Denmark	3	-4	1	9
Germany	-9	15	0	8
Finland	0	2	3	5
United Kingdom	6	-3	-5	4
United States	14	0	-1	4
Belgium	1	1	3	3
Hungary	0	9	0	3
Netherlands	3	5	-1	3

[25] Removing the log transformation does not affect the ranking, as the logarithmic function is a positive monotone transformation.

[26] Each number in the table tells us the positions a country gains (when positive) or losses (when negative) when using the new variables and the HDI(2) with respect to the standard ones.

Sweden	-6	6	3	3
Switzerland	-6	11	0	3
Austria	-2	-2	-2	1
Czech Republic	-2	6	0	1
Greece	-2	-11	0	1
Iceland	1	8	1	1
New Zealand	7	-7	0	1
Slovak Republic	1	5	1	1
Norway	2	-8	1	0
France	-1	4	2	-1
Italy	-13	0	0	-2
Poland	2	17	0	-2
Portugal	1	2	-1	-3
Spain	-3	-13	0	-6
Australia	2	-9	0	-8
Japan	-15	-2	0	-9
Canada	4	-13	-4	-10
Ireland	12	-19	-1	-10
Source: OECD, UN, Human Mortality Database and Eurostat				

A simple way of evaluating the extent of the changes in the ranking is by calculating Spearman's coefficient of correlation. Values close to 1 (resp. -1) indicate a highly positive (resp. a highly negative) correlation, whereas values close to zero indicate lack of correlation. That index, when applied to the ranking generated by the HDI and the HDI(2) yields a result smaller than 0.77, which tells us that we are actually measuring in a different way. Looking for the source of that change in the ranking we find that education is the most important variable in explaining the differences (the Spearma's coefficient tells us that there is practically no correlation whatsoever between both ways of estimating the education index). Material wellbeing is ranked much in the same way, in spite of the huge differences in the coefficients of variation.

Table 8. Spearman's coefficients of correlation. 2007

	Health	Education	Material wellbeing	Human development
Coefficient	0.643	0.175	0.970	0.777

6. The HDI_{2010}

We have presented here a two-fold proposal to improve the HDI while keeping its basic structure. On the one hand, we have defended the need of using a different aggregation criterion (the geometric mean rather that the arithmetic mean) and the introduction of equity elements in the measurement of material wellbeing (which implies using income measures without logs). On the other hand, we have suggested new variables to approach health and education in highly developed countries. We have called HDI(2) the index that includes all those changes and claim that the new index produces much better estimates of human development in highly developed countries.

The 2010 Human Development Report will incorporate some changes concerning the definition of the HDI that are worth discussing with respect to the proposal presented here (we shall refer to this index as the HDI_{2010}). Those changes involve alternative variables to approach education and material wellbeing, and a new normalization scheme. Let us comment on those aspects.

6.1. The 2010 variables

Concerning the health dimension the HDI_{2010} keeps the variable "life expectancy at birth". The use of "life potential", that we propose for highly developed countries, may not be a better alternative for less developed countries. The reason is that advances in life expectancy are very important in those countries, which typically exhibit very high fertility rates and a large share of young population. Therefore, life potential may be misleading when compared with life potential in other countries (particularly with those with high values of life expectancy at birth).

The education component has been substantially reformulated by introducing two variables that substitute the literacy rate and the gross enrolment rate. Those new variables, mean years of schooling (of adults) and school life expectancy (of children), are much more descriptive of the educational achievements. They are very much in the spirit of the variable proposed here for the HDI(2) and we heartly welcome this change. For highly developed countries the variable we have chosen is preferable for the reasons already mentioned and because a single figure is enough to do the job when there is a rather common pattern of compulsory education (typically between 6 and 15 years).

Material wellbeing is approached by means of the Gross National Income per capita (in PPP terms), in logs. The use of the GNI instead of the GDP is a minor refinement and there is nothing to object to that choice. Yet the absence of distributive considerations seems hard to justify when there is information on inequality in most countries. On this point we understand that using inequality adjusted income, either with the GDP or the GNI, is a better alternative because it provides a richer information on the access to material wellbeing and makes it more visible the effect of redistribution policies.

Example 1:[27] South Africa exhibits a per capita GDP that is about 1% higher than that of Panama (5.914 US$ and 5.833 US$, respectively). Yet, the Gini index in South Africa is 37% than in Panama (0.74 and 0.54, respectively). Where a newborn has *ex ante* better chances in life? We strongly believe that ignoring distributive aspects does not help to assess the level of human development. Needless to say, the same reasoning applies to highly developed countries. Take the case of United States and Austria, for instance. Here again USA has a per capita GDP 1% higher than Austria (45.592 US$ and 44.879 US$, respectively), while the inequality is 37% higher (0.4 and 0.29, respectively).

[27] Data from the *Human Development Report 2009*.

6.2. The new normalization procedure

The normalization of partial indices has been modified removing the binding caps (e.g. $100 and $40,000 for income; 99% for adult literacy and 100% for gross enrolment), because an increasing number of countries exceed the upper limits. To keep all indices within the interval [0,1] the actual maximum and minimum values are chosen. This strategy avoids some inconsistencies of the index (now all countries will exhibit values within the prescribed boundaries) at the cost of changing the actual weight of each dimension every year. Note, however, that this change does not really solve the main problem: the dependence of the ranking with respect to the normalization procedure. Solving this problem requires abandoning the linear structure of the index (i.e., to substitute the arithmetic mean by the geometric mean as the aggregation criterion).

Example 2: Imagine that the country with the highest income value in (t-1), let us call it A, significantly increases its income at time *t*, while the rest of the world stays the same. To be precise, suppose that the difference between max and min doubles. The effect on the rest of the countries is that the contribution of the income in *t* dimension is one half of that in (t-1). That may change the relative order of the countries whose original values have not changed at all. That is the case of countries B and C whose partial indices at (t-1) were: B(t-1) = (0.1, 0.2, 0.3) and C(t-1) = (0.2, 0.2, 0.22). The HDI tells us that country B is worse off than country A. Now, due to the change in the scale induced by the income increase of country A, we find the following values: B(t) = (0.05, 0.2, 0.3), C(t-1) = (0.1, 0.2, 0.22). Now country B happens to appear above country C in the ranking, without none of them having experienced any change. This is a major drawback of the proposed normalization when linked to the additive formula.

We propose to use a normalization in which zero is always the min value of the variable. This makes it easier to interpret the normalized values as percentages of the max, whether this is externally given (as assumed here) or it corresponds to the actual max of the year. Note that this normalization together with the multiplicative aggregation formula implies that changes in units (the value of the max) keep both the ranking and the relative distances of the countries. Moreover, the axiomatic derivation of the formula permits one to perform meaningful cardinal comparisons. Therefore, choosing an external max or the endogenous one is mostly a matter of taste. The first choice facilitates making inter-temporal comparisons; the second one always ensures values within limits.

6.3. To log or not to log? That is the question

The HDI_{2010} may use logs in the three variables and not only in that corresponding to material wellbeing. The reason would be the recognition that the effect on human development of one additional unit of a variable does depend on the present level of the variable. The concavity that the log function introduces would capture naturally this fact (even though it flattens the differences between countries).

The rationale for using logs or not is linked to the notion of development we try to capture with our index. If we think of the HDI more as a *welfare measure*, it might be reasonable to keep measuring the three variables in terms of logs (or in terms of any increasing and concave function) as those values can be interpreted as utility measures of an average citizen.[28] If we

[28] That interpretation would also suggest substituting life expectancy at Barth by some index of quality adjusted life years, much in the tradition of health economics.

rather think of the HDI as a capability indicator that provides a reasonable description of the ability of a country to grow, compete and enhance material wellbeing, as we actually do, the use of logs does not seem justified.

One may argue that the HDI_{2010} with logs is ordinally equivalent to our proposed index (letting aside the differences in the chosen variables). This not quite so and the difference is relevant. Taking logs of our index yields the following expression:

$$\log HDI^* = \frac{1}{3}\log(\alpha) + \frac{1}{3}\log(\beta) + \frac{1}{3}\log(\gamma)$$

where α, β, γ are the normalized variables that measure health, education and material wellbeing. Note that here *we first normalize and then take logs* of the normalized variables. In that way the properties of the index concerning the robustness with respect to changes in units is preserved.

The HDI_{2010}, however, adopts the following expression:

$$HDI_{2010} = \frac{1}{3}(\alpha') + \frac{1}{3}(\beta') + \frac{1}{3}(\gamma')$$

where $\lambda' = \frac{\log \lambda - \log(\min \lambda)}{\log(\max \lambda) - \log(\min \lambda)}$, for $\lambda = \alpha, \beta, \gamma$. Here the effect of the normalization on the ranking does not disappear, because *we first take logs and then normalize*.

7. Conclusion

There is a general agreement on the need of revising the HDI, after 20 years of good service. Let us summarize very briefly the main conclusions that derive of this study, formulated in terms of recommendations.

1. It is worth keeping, for the time being, the three traditional dimensions of human development (health, education and material wellbeing). Introducing new dimensions should be part of the research agenda for the immediate future.
2. Using the geometric mean instead of the arithmetic mean, as a way of aggregating the three dimensions into a single indicator, is a clear improvement of the HDI because it makes the index independent on the normalization. Moreover, it is better to normalize the primary variables as a percentage of a maximum value (thus setting the min value equal to zero in all dimensions).
3. Introducing distributive considerations is also conceptually important, even if it does not imply large changes in the ranking. The use of GNI values instead of GDP values is a minor refinement.
4. Defining a specific index for developed countries would improve the descriptive power of the HDI and stimulate the adherence to this standard in OECD countries. On that respect there the variables that approach health and education should be modified along the lines proposed here.

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MULTIDIMENSIONAL SOCIAL EVALUATION: AN APPLICATION TO THE MEASUREMENT OF HUMAN DEVELOPMENT

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This paper deals with the axiomatic derivation of social evaluation indices in a multidimensional context. The resulting evaluation formula is the geometric mean of the egalitarian equivalent values of the different characteristics under consideration. We provide an application to the measurement of human development and compare the results obtained with those corresponding to the standard (additive) index.

1. INTRODUCTION

This paper deals with the construction of social evaluation indices in a multidimensional context. The key purpose is to evaluate the performance of a society as a function of the individual achievements in several dimensions. We follow a non-welfaristic approach, in the sense that our evaluation index is defined directly on the space of *social states* (matrices whose entries represent the agents' achievements in the relevant dimensions), rather than on the joint utility space. More specifically, we aim at contributing to the discussion of the measurement of human development, along the lines laid down by the United Nations (a specially relevant case of this family of evaluation problems).

The Human Development Index (HDI) is an indicator of this type, proposed by the United Nations in order to assess the well-being of a society (United Nations Development Programme 2006–08). Based on Amartya Sen's idea of *functionings and capabilities* (see Sen, 1985), it consists of the arithmetic mean of the partial indices that approach the achievements of the society in three basic dimensions: health, education, and material well-being. Achievements in health are associated with the variable *life expectancy at birth*. Achievements in education

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are approximated by a mixture of two variables, literacy rate and a combined gross enrollment rate (with weights of two-thirds and one-third, respectively). Finally, the achievements in material well-being are measured through the log of the standard per capita GDP. There are also some companion indices that provide measures of some additional aspects (gender and deprivation).

The HDI is simple and intuitive, refers to very relevant aspects of the socio-economic performance, and uses data that are available in most countries. Those features allow for widespread international comparisons that are accessible to non-specialists. That probably explains its popularity and the relevance given by the media to the yearly publication of each new wave of data.

The HDI is certainly a step ahead in the conventional description of the economic performance of a society, as it goes beyond the mere comparison of per capita GDP values. Yet, it is subject to some criticisms, concerning the number and nature of the selected functionings, the choice of the variables that measure those functionings, the lack of theoretical justification of the aggregation formula, or the absence of distributional considerations, among others. The reader is referred to the works of Osberg (1985), Anand and Sen (1994a, 1994b), Philipson and Soares (2001), Osberg and Sharpe (2002), Pinilla and Goerlich (2003), Becker *et al.* (2005), Grimm *et al.* (2008), Hicks (1997), Chakravarty (2003), and Foster *et al.* (2005), for a critical appraisal and some alternative formulations. The last three contributions are those closer to the analysis in this paper.

Chakravarty (2003) follows an axiomatic approach and provides a generalization of the HDI in two steps. First, he gives necessary and sufficient conditions for a characteristic to be measured as a function of its normalized value alone (assuming a compact range, this normalization consists of subtracting the min of the variable and dividing by its range). Then, he provides additional conditions that allow aggregation of those normalized values in terms of an additive formula (the arithmetic mean). The extension of the HDI refers to the possibility of having non-constant rates of substitution between characteristics.

Following a constructive approach, Hicks (1997) and Foster *et al.* (2005) present alternative versions of the HDI that incorporate distributive aspects in the evaluation formula. Hicks (1997) suggests deflating the normalized value in each dimension by a factor involving the Gini index of the corresponding dimension and then aggregating those transformed partial indices using the arithmetic mean. Foster *et al.* (2005) point out that this measure does not satisfy "subgroup consistency" and propose a different measure compatible with such a property. This new measure (actually a family of them) is based on the notion of a *generalized mean*, that may be associated with different specifications of the discount factor, substituting the Gini index by one of the Atkinson's family. The same generalized mean is used to aggregate those partial indices. The introduction of distributive concerns can be interpreted, both in Hicks (1997) and Foster *et al.* (2005), as substituting the original values in each dimension by their corresponding *egalitarian equivalent* ones, suitably defined.

Our focus here is somewhere in the intersection of those contributions. On the one hand, we would like to have a distribution sensitive index for the evaluation of human development. On the other hand, we are concerned with the precise justification of the aggregation formula and, in particular, on the suitability of the arithmetic mean. Indeed, the additive structure of the HDI entails a very particular trade-off between the characteristics, with or without distributive considerations, as it implies assuming full substitutability between all of them (linear indifference curves). That amounts to saying, for instance, that no matter how bad the health state is, it can be compensated with further education or additional income, *at a constant rate*, which is not very natural.

As in Chakravarty (2003), we follow here an axiomatic approach and provide a family of indices in which the marginal rates of substitution between characteristics are not constant. As in Hicks (1997) and Foster *et al.* (2005), our indices are distribution sensitive. More precisely, we provide an axiomatic characterization of an index that consists of the geometric mean of the egalitarian equivalent values of the characteristics. The distributive concern is incorporated in the specification of the egalitarian equivalent values, and admits the Gini index, the (normalized) Theil index, and the Atkinson's family of inequality measures, among others. In the extreme case when distributive aspects do not matter, we obtain a multiplicative version of the standard HDI (i.e. the geometric mean of the mean values).

In order to illustrate the relevance of this approach we consider an empirical application that consists of a comparative study of different formulations of the human development index. We compare the standard human development index with two alternative versions. The first one is similar to the standard index, but without using logs to measure the income. The second alternative is a multiplicative version of the previous one. The results show that choosing the additive or the multiplicative formula matters in two ways. On the one hand, the positions in the raking of human development may change significantly, especially for the medium developed countries. On the other hand, the aggregation procedure substantially affects the distribution of the indicators (in particular the standard HDI shows a much lower dispersion than its multiplicative version).

The rest of the paper is organized as follows. Section 2 presents the formal model and the key results. Section 3 contains the empirical application.

2. The Model and the Results

Let a society consist of n individuals, $N = \{1, 2, \ldots, n\}$, and suppose we want to assess its global performance as a function of the achievements of its members with respect to a set $K = \{1, 2, \ldots, k\}$ of characteristics. The HDI is a case in point, where the characteristics under consideration refer to health, education, and income. A **social state** is a matrix Y with n rows (one for each individual) and k columns (one for each characteristic). The element y_{ij} of matrix Y describes the value of the variable j for individual i. We assume that the values of each of these characteristics vary in the interval [0,1]. This amounts to saying that all variables have been previously normalized in order to make them comparable, independently on the units in which they are originally measured. This normalization procedure can always be done whenever the original values of the variables, z_{ij}, vary in a compact interval $[z_j^0, z_j^*]$, for some non-negative scalars $z_j^0, z_j^* \in \mathbb{R}_+$, with $z_j^0 < z_j^*$ for all $j \in K$. In that case we simply define:

$$y_{ij} = \frac{z_{ij} - z_j^0}{z_j^* - z_j^0},$$

and get the desired normalization. Note that the choice of the upper and lower bounds may not be innocuous, even though we shall not discuss here this subject.[1]

The space of admissible social state matrices is, therefore, $\Omega = [0,1]^{nk}$, that includes the extreme cases Y^0 and Y^*, corresponding to those matrices made out of zeroes and ones, respectively. We denote by $\mathbf{y}_j$ (bold letter) the j-th column of matrix Y that describes the distribution of the j-th characteristic in the population. The vectors $\mathbf{0}_n(j)$, $\mathbf{1}_n(j)$ describe the j-th column of matrices Y^0 and Y^*, respectively. Y_{-j} is an $n \times (k-1)$ matrix obtained from Y by deleting its j-th column. We can therefore write $Y = (Y_{-j}, \mathbf{y}_j)$, assuming that $\mathbf{y}_j$ actually occupies the j-th position in the array of columns.

Consider now the following:

Definition: *A* ***Social Evaluation Index*** *is a continuous single-valued mapping* $I : \Omega \to \mathbb{R}$ *that provides a numerical evaluation of social states.*

A social evaluation index is a continuous function that maps social states into real numbers, in the understanding that higher values of this index correspond to better social states.

Now we consider some properties that introduce value judgments on the evaluation formulae.

The first property, *monotonicity*, establishes that the index increases when all agents in the society strictly improve their achievements.[2]

Monotonicity. For each $Y, Y' \in \Omega$, if $Y >> Y'$ then $I(Y) > I(Y')$.

The second property, *symmetry with respect to the characteristics*, establishes that all characteristics are equally important (recall that all variables have already been normalized). That is, a permutation of the characteristics does not affect the social evaluation. Formally:

Symmetry with respect to the characteristics. For each $Y \in \Omega$, if $\pi_C(Y)$ denotes a permutation of the columns of Y, then $I(\pi_C(Y)) = I(Y)$.

The third property, *normalization*, fixes the scale of the index. It establishes that when the matrix is uniform (i.e. all entries are identical), the index takes on the same value. Formally:

Normalization. Let $Y(\alpha) = \alpha[\mathbf{1}_n(1), \ldots, \mathbf{1}_n(k)]$, for some $\alpha \in [0,1]$. Then, $I(Y(\alpha)) = \alpha$.

It is worth noting that the combination of monotonicity and normalization has strong implications. On the one hand, it implies that the range of I is the interval [0,1], i.e. for all $Y \in \Omega$, $I(Y) \in [0,1]$ with $I_{\min} = I(Y^0) = 0$, $I_{\max} = I(Y^*) = 1$.

[1]See Chakravarty (2003, Th. 1) for an axiomatic deduction of this type of reference variable.
[2]The notation $Y >> Y'$ means that $y_{ij} > y'_{ij}$ for all $i \in N$ and all $j \in K$.

On the other hand, it introduces a cardinality feature in the social evaluation index (it actually implies a unique representation of the index).[3]

The following properties, minimal lower boundedness and separability, are borrowed from the theory of multi-attribute decision-making developed in Bossert and Peters (2000). *Minimal lower boundedness* states that there is no trade-off between characteristics when all members of the society are at their worst level in one of them. In other words, if for some characteristic $j \in K$ we have $y_{ij} = 0$, for all $i \in N$, then the social evaluation index I takes on its minimum value. Therefore, for a characteristic to provide a positive contribution to the social performance, at least one individual in society should be above that minimum level.[4] Formally:

Minimal lower boundedness. For all $Y, Y' \in \Omega$, all $j \in K$, $I(Y) \geq I\left(Y'_{-j}, \mathbf{0}_n(j)\right)$.

Clearly, this property, together with monotonicity and normalization, implies that $I(Y_{-j}, \mathbf{0}_n(j)) = 0$, for all $j \in K$, all $Y \in \Omega$.

Separability is a property closely related to the preferential independence axiom in utility theory (Keeney and Raiffa, 1976, ch. 3). It establishes that if social state Y is considered at least as good as social state Y', when there is a common value of a characteristic (both have an identical column $\mathbf{y}_j$), then this relation holds for all common values of this column. For this property to be compatible with minimal lower boundedness, it is only required on those matrices with strictly positive entries. Formally:

Separability. For each $Y, Y' \in \Omega$ with $Y, Y' >> Y^0$, and each $j \in K$,

$$I\left(Y_{-j}, \mathbf{y}_j\right) \geq I\left(Y'_{-j}, \mathbf{y}_j\right) \Rightarrow I\left(Y_{-j}, \mathbf{y}'_j\right) \geq I\left(Y'_{-j}, \mathbf{y}'_j\right).$$

Given a social evaluation index I satisfying the aforementioned properties, and a social state matrix Y, we denote by $\xi\left(Y_{-j}, \mathbf{y}_j\right) \in \mathbb{R}$ the **egalitarian equivalent value** associated with the distribution of the j-th characteristic in Y. That is, $\xi(Y_{-j}, \mathbf{y}_j)$ is implicitly defined by the following equation:

$$I(Y) = I\left(Y_{-j}, \mathbf{1}_n(j)\xi\left(Y_{-j}, \mathbf{y}_j\right)\right).$$

When $\xi(Y_{-j}, \mathbf{y}_j)$ is independent on Y_{-j} (that is, $\xi\left(Y_{-j}, \mathbf{y}_j\right) = \xi\left(Y'_{-j}, \mathbf{y}_j\right)$), for all admissible Y'_{-j}, we shall simply write: $\xi(\mathbf{y}_j)$.[5]

The next result is obtained:

Theorem: *A social evaluation index satisfies monotonicity, symmetry with respect to the characteristics, normalization, minimal lower boundedness, and separability, if and only if it takes the form*

[3]This relation is formally stated in a lemma in the Appendix.

[4]Note that the strength of this requirement partly depends on how we define the minimum value of the characteristic under consideration. For instance, if $y_{ij} = 0$ describes an absolute minimal value (e.g. income below subsistence), then this property is automatically fulfilled. When $y_{ij} = 0$ represents some conventional reference level, then the requirement implicitly says that this level is the minimum admissible in order to allow other characteristics to be taken into account.

[5]We show (see step 3 of the proof of Theorem 1) that, under our assumptions, the egalitarian equivalent value is well defined and actually independent of Y_{-j}.

$$I(Y) = \prod_{j \in K} \left(\xi(\mathbf{y}_j)\right)^{1/k},$$

where $\xi(\mathbf{y}_j)$ *is the egalitarian equivalent value of* $\mathbf{y}_j$. *Moreover, all these properties are logically independent.*

(The proof is given in the Appendix)

This theorem says that assuming the five properties above amounts to measuring social states as the geometric mean of the egalitarian equivalent values of the corresponding characteristics.

The theorem identifies a family of multiplicative indices, $\mathbf{I}_M$, rather than a single one. To achieve a fully closed formula we have to make precise the content of the egalitarian equivalent values, $\xi(.)$. This is a notion that expresses our concern for equity in the distribution. Following the standard approach in the theory of economic inequality, under standard conditions,[6] we can define

$$\xi(\mathbf{y}_j) = \mu(\mathbf{y}_j)[1 - f(\mathbf{y}_j)],$$

where $\mu(\mathbf{y}_j)$ is the mean of the j-th characteristic and $f(.)$ is an index of inequality. We can then modulate our concern for equality via the specification of the inequality index used to define the egalitarian equivalent values.

Note that not all of the usual indices are suitable for this purpose. This is so because each $\xi(\mathbf{y}_j)$ must be increasing in $\mathbf{y}_j$ and take on values in the interval [0,1]. That requires $f(\mathbf{y}_j)$ to range also in that interval. Natural candidates to be considered are, therefore, the Gini index, the normalized (first) index of Theil and the Atkinson's family of inequality indices.

Depending on the chosen inequality measure, the social evaluation index will exhibit more precise properties (e.g. population replication, subgroup consistency, etc.). Indeed, making use of the theory of inequality measurement one can fix the inequality index out of the properties one is willing to get (see, for instance, Foster *et al.*, 2005).

Observe that the inequality measure in this formulation applies to the normalized values $y_{ij} = (z_{ij} - z_j^0)/(z_j^* - z_j^0)$, with $z_{ij}, \in [z_j^0, z_j^*]$, for some non-negative scalars $z_j^0, z_j^* \in \mathbb{R}_+$. If $z_j^0 = 0$ for all $j \in K$, then any relative inequality index will measure, precisely, the distribution of the original values.

A special case is that in which we are not concerned with equality. This is precisely the case of the standard human development index. The next property, *distributional neutrality*, introduces this idea. It says that the average value of a characteristic is sufficient information to calculate the index. Formally:

Distributional neutrality in the *j*-th characteristic. For each $Y \in \Omega$, $I(Y) = I(Y_{-j}, \mathbf{1}_n(j)\mu(\mathbf{y}_j))$.

Trivially, when I is distributionally neutral in the j-th characteristic, $\xi(\mathbf{y}_j) = \mu(\mathbf{y}_j)$. Furthermore, as our indices satisfy *symmetry with respect to the*

[6]That basically requires the index to be anonymous (names do not matter) and quasi-concave (redistribution is good).

characteristics, whenever $I \in \mathbf{I}_M$ is distributionally neutral w.r.t. some characteristic, it will also be neutral w.r.t all of them.[7] Therefore, the following straightforward result is obtained:

Corollary: *An index $I \in \mathbf{I}_M$ satisfies distributional neutrality in some characteristic j, if and only if it takes the form:*

$$I(Y) = \prod_{j \in K} [\mu(\mathbf{y}_j)]^{1/k}.$$

This result says that assuming distributional neutrality amounts to taking the geometric mean of the mean values of each characteristic as the right indicator. The formula obtained in the Corollary is one of the limit cases in the family of indices analyzed in Foster *et al.* (2005).

Note that, even though the index in the Corollary discards the information concerning the inequality in the distribution of each characteristic, it penalizes countries with uneven mean achievements in the different dimensions, reflecting the view that there is some complementarity between characteristics rather than full substitutability. Also observe that this index satisfies the principle of population replication (i.e. the index does not change when we replicate the population). Therefore, we can apply this index to societies with different population size and compare the performance of those societies with respect to the same set of variables.

3. A Case Study

We devote this section to illustrating the differences between the human development index proposed by the United Nations and our multiplicative index. We follow the method of the Human Development Report 2006, using its data for health, education, and income corresponding to 177 countries. Even though our formulation allows for the introduction of distributive considerations, we shall develop our analysis here in terms of the formula in the Corollary. The reason is twofold: on the one hand, to facilitate the comparison on the effect of modifying the aggregation criterion; on the other hand, due to the fact that the variables related to health and education are directly average constructs, without much reference to individual values, so that their dispersion in the population cannot be clearly established (see below).

The health variable is measured through the life expectancy at birth,

$$m(\mathbf{h}) = \frac{\text{life expectancy at birth} - 25}{85 - 25}.$$

[7] By the same token, if we introduce distributive considerations with respect to a particular dimension in terms of a given inequality measure, the same inequality index should be applied to all dimensions.

The education variable is measured through a combination of the adult literacy rate and the gross enrollment ratio, with weights of two-thirds and one-third, respectively,

$$m(\mathbf{e}) = \frac{\left(\frac{2}{3}\text{adult literacy rate} + \frac{1}{3}\text{gross enrollment ratio}\right) - 0}{100 - 0}$$
$$= \frac{2}{3}\frac{\text{adult literacy rate} - 0}{100 - 0} + \frac{1}{3}\frac{\text{gross enrollment ratio} - 0}{100 - 0}.$$

The income variable is measured through the log of the GDP per capita

$$m(\mathbf{y}) = \frac{\log(\text{GDP per capita}) - \log 100}{\log 40{,}000 - \log 100}.$$

Alternatively, we also consider the partial indicator $m'(\mathbf{y})$ where we dispense with the logs:

$$m'(\mathbf{y}) = \frac{\text{GDP per capita} - 100}{40{,}000 - 100}.$$

With these four partial indicators we can define a collection of human development indices, the first of which corresponds to the United Nations original proposal; the last one is the index characterized in the Corollary. The other index represents an intermediate step that is useful to clarify the nature of the differences between the United Nations HDI and our proposal. That is:

UN Human Development Index. $\text{UNHDI} = \frac{1}{3}(m(\mathbf{h}) + m(\mathbf{e}) + m(\mathbf{y}))$.

The second index also has an additive structure, but the income indicator is taken without logs.

Additive Human Development Index. $\text{AHDI} = \frac{1}{3}(m(\mathbf{h}) + m(\mathbf{e}) + m'(\mathbf{y}))$.

The third index is the multiplicative version of the previous one (see the Corollary).

Multiplicative Human Development Index. $\text{MHDI} = (m(\mathbf{h}) \cdot m(\mathbf{e}) \cdot m'(\mathbf{y}))^{\frac{1}{3}}$.

We plot on the horizontal axis of Figure 1 the 177 countries in the UN reports, arranged in decreasing order according to the UNHDI. The vertical axis shows the values of these three indices. Peaks correspond to changes in the order for the indices with respect to UNHDI. This figure illustrates well that these indices are not ordinally equivalent.[8] Indeed, some countries jump substantially in the ranking (Colombia, for instance, loses 11 positions). The difference in the ranking between UNHDI and AHDI is due to the re-scaling effect derived from

[8]Note that this happens not only when we move from an additive formulation to a multiplicative one, but also when we use the income variable without logs.

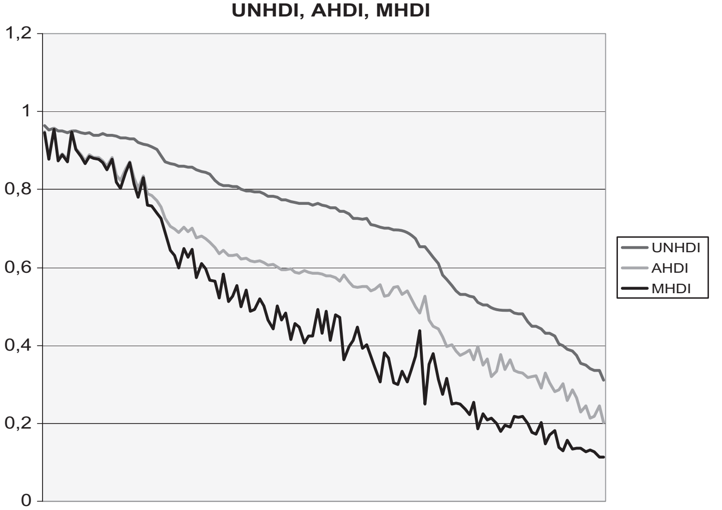

Figure 1. Absolute Values of the Different Indices (2006)

taking out the logs in the income measure, because that change alters the weight with which this variable enters the index. The difference between the AHDI and the MHDI ranking derives from the effect of the differences in the partial indices for each country: for a constant sum, the more equal the partial indices the higher the value of its product (this also explains the fact that the additive indices dominate the multiplicative one).

The cardinal information that these indices provide is also substantially different. A first approximation comes from the analysis of their dispersion, approximated by some free scale measure. The *coefficient of variation* is a standard measure of this type. As one would expect, the dispersion is larger for the MHDI than for the UNHDI. The coefficient of variation of the UNHDI is 0.26, whereas the value for the MHDI is 0.53 (more than twice that of the UNHDI). This suggests that the UNHDI is somehow hiding the differences between countries. On the other hand, the coefficient of variation for the AHDI is 0.35, pointing out that the difference in the dispersion is mainly due to the aggregation procedure and not to the use of logs.

Figure 2 presents the distribution of these indices, normalizing to one the highest value achieved by a country with each index. Note that here the same point in the horizontal axis may correspond to different countries, depending on the chosen index. The graphic shows a rather different picture of the distribution of the development levels, depending on the indicator. For those countries with higher values of the UNHDI the observed difference is mainly due to the measurement in logs of the income levels (which is anyway relevant for the whole distribution). The choice of an additive or a multiplicative formula becomes much more important for those countries in the mid range of the distribution and it is also substantial for those with lower values. A way of illustrating these differences is by noting that the least developed countries represent 32 percent

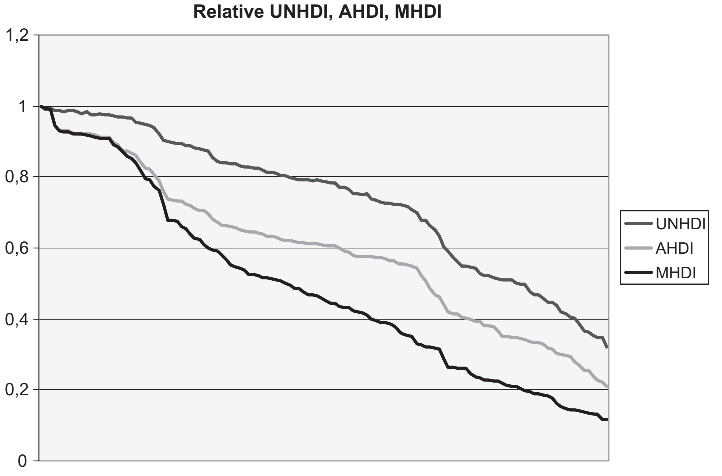

Figure 2. Relative Values of the Different Indices (2006)

of the most developed ones, according to the UNHDI, whereas this figure is 11 percent according to the MHDI. In terms of the correlations, the three indices are quite related. The correlations between the AHDI and UNHDI, and the AHDI and MHDI are 0.98 in both cases; while the correlation between the UNHDI and the MHDI is 0.94.

Table 1 provides the data with which these figures have been constructed, for a representative sample of 50 countries with different development levels. The table gives the values of the partial indices measuring the achievements in health, education, and material well-being, and the resulting human development indices, as well as the changes in the ranking of the MHDI with respect to the UNHDI. A positive number in this last column indicates that the country has gained positions in the MHDI with respect to the UNHDI.

In summary, this application shows that the choice of the aggregation formula for the partial indices matters as it affects both the ranking of the countries and the dispersion of their values. The standard HDI reduces the observed inequality between countries both due to the use of logs in the measurement of income and to the additive aggregation procedure. The multiplicative formula proposed here presents some advantages. First, it is theoretically well justified. Second, it does not assume constant rates of substitution between characteristics. And third, it allows for distributive considerations. The discussion above suggests that dealing properly with the distributive aspects calls for a modification of the way in which the United Nations measures the achievements in health and education.[9] This topic is left for future research.

[9]See, however, the original approach in Grimm *et al.* (2008), that allows for the introduction of distributive aspects recurring to the computation of the index by income groups.

TABLE 1

Human Development Indices for a Sample of Countries (2006)

	Means				HDI			Ranking Changes
	$m(\mathbf{h})$	$m(\mathbf{e})$	$m(\mathbf{y})$	$m'(\mathbf{y})$	UNHDI	AHDI	MHDI	
High human development								
Ireland	0.88	0.99	1.00	1.00	0.957	0.957	0.955	−2
United States	0.88	0.97	1.00	1.00	0.950	0.950	0.949	−3
Canada	0.92	0.97	0.96	0.79	0.950	0.892	0.889	1
Belgium	0.90	0.98	0.96	0.79	0.947	0.889	0.885	−1
Netherlands	0.89	0.99	0.96	0.79	0.947	0.889	0.885	−1
Austria	0.90	0.96	0.96	0.79	0.940	0.882	0.879	−5
United Kingdom	0.89	0.97	0.96	0.79	0.940	0.882	0.879	−5
Denmark	0.87	0.99	0.96	0.79	0.940	0.882	0.878	−5
Sweden	0.92	0.98	0.95	0.74	0.950	0.880	0.874	6
Japan	0.95	0.94	0.95	0.74	0.947	0.877	0.871	4
France	0.91	0.97	0.95	0.74	0.943	0.873	0.868	4
Finland	0.89	0.99	0.95	0.74	0.943	0.873	0.867	4
Italy	0.92	0.96	0.94	0.70	0.940	0.859	0.851	1
Germany	0.90	0.96	0.94	0.70	0.933	0.852	0.845	−1
Spain	0.91	0.98	0.92	0.62	0.937	0.836	0.820	3
Greece	0.89	0.97	0.90	0.55	0.920	0.803	0.779	2
Portugal	0.87	0.96	0.88	0.49	0.903	0.772	0.740	0
Medium human development								
Brazil	0.76	0.88	0.74	0.21	0.793	0.616	0.519	−4
Thailand	0.75	0.86	0.73	0.20	0.780	0.602	0.502	−8
Colombia	0.79	0.86	0.72	0.18	0.790	0.612	0.501	−2
Turkey	0.73	0.81	0.73	0.20	0.757	0.579	0.488	−18
Iran, Islamic Rep. of	0.76	0.75	0.72	0.18	0.743	0.565	0.472	−17
China	0.78	0.84	0.68	0.14	0.767	0.588	0.456	7
Peru	0.75	0.87	0.67	0.14	0.763	0.585	0.446	8
South Africa	0.37	0.80	0.79	0.28	0.653	0.484	0.437	−28
Paraguay	0.77	0.86	0.65	0.12	0.760	0.584	0.431	5
Philippines	0.76	0.89	0.64	0.11	0.763	0.588	0.425	12
Ecuador	0.82	0.86	0.61	0.09	0.763	0.591	0.405	17
Egypt	0.75	0.73	0.62	0.10	0.700	0.527	0.380	−3
Namibia	0.37	0.79	0.72	0.18	0.627	0.448	0.378	−16
Morocco	0.75	0.54	0.63	0.11	0.640	0.466	0.351	−8
Honduras	0.72	0.77	0.56	0.07	0.683	0.520	0.337	1
Bolivia	0.66	0.87	0.55	0.07	0.693	0.532	0.334	5
India	0.64	0.61	0.58	0.08	0.610	0.443	0.313	−4
Mauritania	0.47	0.49	0.49	0.04	0.483	0.335	0.218	−9
Guinea	0.48	0.34	0.51	0.05	0.443	0.290	0.202	−11
Senegal	0.52	0.39	0.47	0.04	0.460	0.316	0.200	−5
Low human development								
Kenya	0.37	0.69	0.41	0.03	0.490	0.362	0.190	5
Cote d'Ivoire	0.35	0.46	0.46	0.04	0.423	0.282	0.181	−6
Rwanda	0.32	0.61	0.42	0.03	0.450	0.320	0.177	2
Nigeria	0.31	0.63	0.41	0.03	0.450	0.322	0.173	3
Benin	0.49	0.40	0.40	0.03	0.430	0.305	0.170	1
Tanzania, U. Rep. of	0.35	0.62	0.32	0.01	0.430	0.328	0.147	3
Zambia	0.21	0.63	0.37	0.02	0.403	0.287	0.139	2
Ethiopia	0.38	0.40	0.34	0.02	0.373	0.266	0.136	−1
Central African Republic	0.24	0.42	0.40	0.03	0.353	0.228	0.136	−2
Burundi	0.32	0.52	0.32	0.01	0.387	0.285	0.134	2
Burkina Faso	0.38	0.23	0.41	0.03	0.340	0.212	0.133	−2
Malawi	0.25	0.64	0.31	0.01	0.400	0.301	0.129	7
Niger	0.33	0.26	0.34	0.02	0.310	0.202	0.113	1

Appendix

Lemma. An index that satisfies monotonicity and normalization together is cardinal.

Proof. Let I be a monotonic and normalized index. Now, consider a continuous and increasing function $G:\mathbb{R}\rightarrow\mathbb{R}$, and take the transformation of a social evaluation index I, $(G\circ I)$. One may wonder whether $(G\circ I)$ is also a suitable index to evaluate social states, and fulfils those properties. Since $(G\circ I)$ must satisfy normalization, $(G\circ I)(Y(\alpha))=\alpha$. On the other hand,

$$(G\circ I)(Y(\alpha))=G(I(Y(\alpha)))=G(\alpha).$$

Therefore, $G(\alpha)=\alpha$, and G is the identity function. This implies not only that I is cardinal but also that this representation is unique, as $I(Y^0)=0$, $I(Y^*)=1$, and there is no degree of freedom left.

Proof of the Theorem

It is not difficult to check that such indices satisfy the properties. We show here the converse. Let I be an index that satisfies all the five properties.

Step 1. Multiplicative structure of the index. In this step we show that an index that satisfies monotonicity, normalization, minimal lower boundedness, and separability is multiplicative. Indeed, by *monotonicity* and *normalization* we can write:

$$0=I(Y^0)\leq I(Y)\leq I(Y^*)=1,\quad \text{for all } Y\in\Omega.$$

Take an arbitrary characteristic, $j\in K$. By *separability*, and *cardinality* (obtained in application of the lemma), since $\mathbf{y}_j$ is independent of Y_{-j}, there exist real valued mappings u,v such that:[10]

$$I(Y)=u(\mathbf{y}_j)+v(\mathbf{y}_j)I(Y_{-j},\mathbf{1}_n(j)).$$

By letting $Y=(Y^0_{-j},\mathbf{y}_j)$, *minimal lower boundedness* implies $u(\mathbf{y}_j)=0$, for all $\mathbf{y}_j$. Now, letting $Y=(Y^*_{-j},\mathbf{y}_j)$, we get $v(\mathbf{y}_j)=I(Y^*_{-j},\mathbf{y}_j)$ and, consequently,

$$I(Y)=I(Y^*_{-j},\mathbf{y}_j)\cdot I(Y_{-j},\mathbf{1}_n(j)).$$

Define now the function $\hat{I}(Y_{-j}):=I(Y_{-j},\mathbf{1}_n(j))$, that satisfies *separability* and *minimal lower boundedness*, and apply to $\hat{I}$ the argument above for some characteristic $h\neq j$. That is, by *separability* and *cardinality*, there are two functions $\hat{u}$ and $\hat{v}$ such that

$$\hat{I}(Y_{-j})=\hat{u}(\mathbf{y}_h)+\hat{v}(\mathbf{y}_h)\hat{I}(Y_{-jh},\mathbf{1}_n(h)).$$

[10]See Keeney and Raiffa (1976, ch. 5, 6). Note that, in principle, both u and v may depend on the characteristic j considered. Yet we shall not make specific this feature in order to save notation.

By letting $Y_{-j} = (Y_{-jh}, \mathbf{y}_h)$, $Y^*_{-j} = (Y^*_{-jh}, \mathbf{y}_h)$, we get

$$\hat{I}(Y_{-j}) = \hat{I}(Y^*_{-jh}, \mathbf{y}_h) \cdot \hat{I}(Y_{-jh}, \mathbf{1}_n(h)).$$

Since, by definition, $\hat{I}(Y_{-j}) = I(Y_{-j}, \mathbf{1}_n(j))$, we have that

$$I(Y_{-j}, \mathbf{1}_n(j)) = I(Y^*_{-h}, \mathbf{y}_h) \cdot I(Y_{-jh}, \mathbf{1}_n(j), \mathbf{1}_n(h)).$$

Replacing $I(Y_{-j}, \mathbf{1}_n(j))$ in $I(Y)$,

$$\begin{aligned} I(Y) &= I(Y^*_{-j}, \mathbf{y}_j) I(Y_{-j}, \mathbf{1}_n(j)) \\ &= I(Y^*_{-j}, \mathbf{y}_j) \cdot I(Y^*_{-h}, \mathbf{y}_h) \cdot I(Y_{-jh}, \mathbf{1}_n(j), \mathbf{1}_n(h)). \end{aligned}$$

By repeating this procedure for all characteristics, we arrive at:

$$I(Y) = I(Y^*_{-1}, \mathbf{y}_1) \cdot I(Y^*_{-2}, \mathbf{y}_2) \cdot \ldots \cdot I(Y^*_{-k}, \mathbf{y}_k).$$

Therefore, if we define

$$F_j(\mathbf{y}_j) = I(Y^*_{-j}, \mathbf{y}_j),$$

we get:

$$I(Y) = \prod_{j \in K} F_j(\mathbf{y}_j),$$

with $F_j(\mathbf{0}_n) = 0$ and $F_j(\mathbf{1}_n) = 1$.

Step 2. Adding symmetry. By *symmetry with respect to the characteristics*, $F_j(.) = F_h(.) = F(.)$, for all $j,h \in K$. Therefore, we can write:

$$I(Y) = \prod_{j \in K} F(\mathbf{y}_j).$$

Step 3. Existence of the egalitarian equivalent value. We now see that the egalitarian equivalent value, as it is defined in Section 2, exists, and it does not depend on Y_{-j}. As a preliminary stage, we show that if $(Y^*_{-j}, \mathbf{y}_j) \in \Omega$ then there exists $\xi(\mathbf{y}_j) \in [0,1]$ such that $I(Y^*_{-j}, \mathbf{y}_j) = I(Y^*_{-j}, \mathbf{1}_n \xi(\mathbf{y}_j))$. To do so, define a function $g: [0,1] \to \mathbb{R}$ as follows: $g(\xi) = I(Y^*_{-j}, \mathbf{1}_n \xi)$. Such a g is a continuous function, with $g(0) = I(Y^*_{-j}, \mathbf{0}_n) = 0$, and $g(1) = I(Y^*_{-j}, \mathbf{1}_n) = 1$. Since $0 \le I(Y^*_{-j}, \mathbf{y}_j) \le 1$, the mean value theorem guarantees that there exists some $\xi(\mathbf{y}_j)$ such that $I(Y^*_{-j}, \mathbf{y}_j) = g(\xi(\mathbf{y}_j)) = I(Y^*_{-j}, \mathbf{1}_n \xi(\mathbf{y}_j))$.

Now we show that the egalitarian equivalent value exists for any Y_{-j}. Indeed, let $\xi(\mathbf{y}_j) \in [0,1]$ be such that $I(Y^*_{-j}, \mathbf{y}_j) = I(Y^*_{-j}, \mathbf{1}_n \xi(\mathbf{y}_j))$. We prove that such a value also satisfies $I(Y_{-j}, \mathbf{y}_j) = I(Y_{-j}, \mathbf{1}_n \xi(\mathbf{y}_j))$. By definition,

$$F(\mathbf{y}_j) = I(Y^*_{-j}, \mathbf{y}_j) = I(Y^*_{-j}, \mathbf{1}_n\xi(\mathbf{y}_j)) = F(\mathbf{1}_n\xi(\mathbf{y}_j)).$$

Then,

$$\begin{aligned} I(Y_{-j}, \mathbf{y}_j) &= \prod_{h\neq j} F(\mathbf{y}_h)\cdot F(\mathbf{y}_j) \\ &= \prod_{h\neq j} F(\mathbf{y}_h)\cdot F(\mathbf{1}_n\xi(\mathbf{y}_j)) \\ &= I(Y_{-j}, \mathbf{1}_n\xi(\mathbf{y}_j)). \end{aligned}$$

Step 4. Closing the formula. The previous result indicates that the egalitarian equivalent value $\xi(\mathbf{y}_j)$ obtained in previous step does not depend on the distribution of the other characteristics. Making use of *normalization* once more, and the definition of egalitarian equivalent value above, we can write:

$$\begin{aligned} \xi(\mathbf{y}_j) &= I[\mathbf{1}_n\xi(\mathbf{y}_j), \ldots, \mathbf{1}_n\xi(\mathbf{y}_j)] \\ &= [F(\mathbf{1}_n\xi(\mathbf{y}_j))]^k. \end{aligned}$$

Therefore,

$$F(\mathbf{y}_j) = [\xi(\mathbf{y}_j)]^{1/k},$$

for all $j \in K$, and the result follows. Notice, furthermore, that whenever $\xi(\mathbf{y}_j) \neq 0$, we can also guarantee that $\xi(\mathbf{y}_j)$ is unique.

Step 5. Separation of the properties. In order to see that all properties in Theorem 1 are independent, we provide examples of indices satisfying all but one property.

$$I_1(Y) = \min_{i,j} y_{ij}.$$

$$I_2(Y) = HDI(Y) = \frac{1}{k}\sum_{j=1\ldots k} \mu(y_j).$$

$$I_3(Y) = \prod_{j=1}^{k} \mu(\mathbf{y}_j)^{\alpha_j}$$, with $\alpha_j > 0$ for all j, $\Sigma_j\alpha_j = 1$, and for some l,m, $\alpha_l \neq \alpha_m$.

$$I_4(Y) = \prod_{j=1}^{k} \mu(\mathbf{y}_j).$$

$$I_5(Y) = \prod_{j=1}^{k} \left[\mu(\mathbf{y}_j)^{1/k} - \text{range}(\mathbf{y}_j)^k\right].$$

In the following table we summarize the properties satisfied by these indices:

	Mon	Sym	Norm	MLB	Sep
I_1	Yes	Yes	Yes	Yes	No
I_2	Yes	Yes	Yes	No	Yes
I_3	Yes	No	Yes	Yes	Yes
I_4	Yes	Yes	No	Yes	Yes
I_5	No	Yes	Yes	Yes	Yes

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